Calc 12 - Chp 3.6

Note Title 2013-09-23

Chain Rule - used for composite fns.

Intro to Composite Fns – take the output of one fn and use it as an input to another fn. This can happen more than once.

This is common in business.

$f(g(x)) = (f \circ g)(x)$
"f of g of x"

$x = \text{Cost}(p)$
$y = \text{DistributionPrice}(x)$
$z = \text{WholesalePrice}(y)$
$u = \text{RetailPrice}(z)$
$v = \text{Tax}(u)$
$v = T(R(W(D(C(p)))))$
$= (T \circ R \circ W \circ D \circ C)(p)$

Chain Rule: $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

Simple

eg) $f(x) = (x^3)^5$ $\Rightarrow = x^{15}$
$= 15x^{14}$

simplify then differentiate.

Medium:

eg) $f(x) = \sin(x^3)$

chain rule solves this; can't simplify first!

Power Chain Rule: $f(x) = [g(x)]^n$

eg) $f(x) = \sin^2 x$

or

$f(x) = \sin x \cdot \sin x$

Hard

eg) $f(x) = \sin(\sin x^2)$

Another way to view chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
(Substitution)

$y = \cos(x^4)$

The case for radians:

$f(x) = \sin x$ (in degrees)
$= \sin\left(\frac{\pi x}{180}\right)$

#34 $f(u) = 1 - \frac{1}{u}$

$g(x) = \frac{1}{1-x}$

#36 $f(u) = u + \frac{1}{\cos^2 u}$

$g(x) = \pi x$

HW: pp. 146-147: 1-33 (odds), 32

Challenge: 35, 37

Calc 12 - Chp 3.7

Implicit Differentiation - Some equations are hard to express in terms of y. So an easier way to find the derivative is to use implicit differentiation.

eg) $x^3 + y^2 = 15$. Find dy/dx

differentiate each term wrt x

y is a function of x so use chain rule

isolate $\frac{dy}{dx}$

Very General Steps for Implicit Differentiation:

1. differentiate each term wrt x
2. collect dy/dx terms on LHS.
3. factor out dy/dx
4. isolate dy/dx.

Note: can use y' instead of dy/dx

---

eg) find y' of $x^2y^2 + x^3 + y^4 = 20$

eg) Find y'' of first example: $y' = \frac{-3x^2}{2y}$

---

eg) With trig: Find y' of $7x^2 + 3x\cos y = xy$

HW: p. 155: 3, 9, 11, 17, 19, 23 - 29 (odds), 28
Challenge: 22

Calc 12 - Chp 3.8

Note Title 2013-09-23

Derivatives of Inverse Functions.

p. 158 Theorem 3 - Derivative of Inverse Fns.

If is differentiable at every point of an interval and is never on , then has an inverse and is differentiable at every point of the interval .

Big picture: If we can differentiate, it stands that we can take the reciprocal of the derivative.

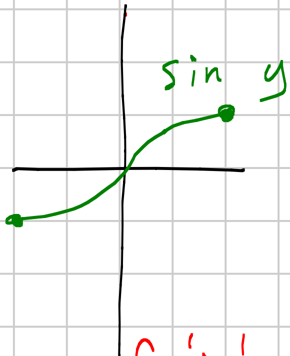
eg) $y = \sin^{-1} x$

$\sin y = x$ inverse

$\frac{d}{dx} \sin y = \frac{d}{dx} x$ implicit diff

$y' \cos y = 1$ differentiate

$y' = \frac{1}{\cos y}$ isolate y', but must solve in terms of 'x'



p. 161

Inverse Function – Inverse Cofunction Identities

$\cos^{-1} x = \pi/2 - \sin^{-1} x$

$\cot^{-1} x = \pi/2 - \tan^{-1} x$

$\csc^{-1} x = \pi/2 - \sec^{-1} x$

Calculator Conversion Identities

$\sec^{-1} x = \cos^{-1} (1/x)$

$\cot^{-1} x = \pi/2 - \tan^{-1} x$

$\csc^{-1} x = \sin^{-1} (1/x)$

Derivatives of Inverse Trig Fns.

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\csc^{-1}x = \frac{-1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\sec^{-1}x = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2} \qquad \frac{d}{dx}\cot^{-1}x = \frac{-1}{1+x^2}$$

eg) $f(x) = \sin^{-1}(x^3)$ Find $f'(.2)$

Let $u = x^3$:

$\frac{du}{dx} = 3x^2$

eg) $f(x) = \cot^{-1}\frac{1}{x} - \tan^{-1}x$ Find $f'(x)$

$f'(x) =$

eg) $f(t) = \dfrac{1}{\cos^{-1}(3t^2)}$ find $f'(.4)$

HW: p. 162: 1–19 (odds), 10

Challenge: $y = \cos^{-1} x$. Derive y' using implicit differentiation

21

Calc 12 - Chp 3.9

Note Title 2013-09-23

Derivatives of Exponential and Logarithmic Fns

Exponential Functions - looks like powers.
However, there is a major difference; the variable is in exponent rather than the base and the base must be

The domain for exp fns is all reals.
Range for untransformed exp fn is $y > 0$
Untransformed exp fns are ALWAYS INCREASING!

Definition of a Logarithm - The logarithm of a number is an exponent. $\log_b c$ is the power to which is raised to get . The base of the logarithm is the same as the base of the power.
When $\log_b c = a$, then , where

Exponentials and Logarithms are inverses of each other!
eg) $4^3 = 64$ $\quad \log_4 64 = 3$

$2^4 \cdot 2^5 = 2^9$ $\quad \log 2^4 + \log 2^5$
$4 \cdot 8 = 32$ $\quad \log 4 + \log 8$
$\log_a x + \log_a y = \log_a xy$

$\frac{3^7}{3^4} = 3^3$ $\quad \log 3^7 - \log 3^4 =$
$28/7 = 4$ $\quad \log 28 - \log 7 =$
$\log_a x - \log_a y = \log_a (x/y)$

$(5^2)^4 = 5^8$ $\quad \log (5^2)^4 =$
$\log 6^3 =$
$\log_a x^k = k \log_a x$

$\log_a 1 = 0$

$\log_a a = 1$

$\log_a a^b = b$

change of base

$\log_b a = \frac{\log_c a}{\log_c b}$
$= \frac{\log a}{\log b}$

e^x is a most interesting fn.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots$$

$$e^x = \lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n$$

$$\frac{d}{dx} e^x = e^x$$

Property: $\lim_{h\to 0} \frac{e^h - 1}{h} = 1$ or $\lim_{x\to 0} \frac{e^x - e^0}{x - 0} = 1$ another way to look at it

$f(x) = e^x$

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

difficult? No.

Chain Rule:

$$\frac{d}{dx} e^u = e^u \frac{du}{dx}$$

Derivative of a^x, $a > 0$ & $a \neq 1$

$\frac{d}{dx} a^x =$

exp rules.

use chain rule.

Chain Rule: $\frac{d}{dx} a^u = a^u \ln a \frac{du}{dx}$ $a > 0$ & $a \neq 1$

Derivative of $\ln x$, $x > 0$

(1) $y = \ln x$

exponentiate both sides.

diff wrt x

isolate y

substitute from (1)

Chain Rule: $\frac{d}{dx} \ln u = \frac{1}{u} \frac{du}{dx}$

Derivative of $\log_a x$, $a > 0$, $a \neq 1$, $x > 0$

$\frac{d}{dx} \log_a x =$

exp rules.

Chain Rule: $\frac{d}{dx} \log_a u = \frac{1}{u \ln a} \frac{du}{dx}$ $a > 0$ & $a \neq 1$

eg) $y = e^{-x/4}$ Find y'

eg) $y = 9^{-x}$ Find y'

eg) $y = (\ln x)^2$

eg) $y = \log_{10}\sqrt{x+1}$

eg) $y = x^{\tan x}$, $x > 0$

HW: p. 170: 1 - 37 (every other odd), 28
Challenge: 43
Hint: Take ln & implicit differentiation.

Review for Test: pp. 172 - 175: 11 - 25 (odds), 35 - 41 (odds), 69, 70.