Calc 12 - Chp 1 Review/Ref Sheet

Note Title 2013-09-23

Slope $= m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Slope Intercept Form: $y = mx + b$
Point-Slope Form: $y - y_1 = m(x - x_1)$
} Start at point & use slope

General Form: $Ax + By + C = 0$
Standard Form: $Ax + By = C$
} calculate intercepts & draw line

Parallel Lines: $m_1 = m_2$ $\quad L_1 \parallel L_2$
Perpendicular Lines: $m_1 = -1/m_3$ $\quad L_1 \perp L_3$
Zero slope: $m_4 = 0$ $\quad L_4$
No slope: m_5 is undefined

Function (formal definition): A fn f is a rule that assigns to each element x in a set A exactly one element, called f(x), in a set B.
Recall: Use vertical line test.

Domains & Ranges:
Recall notation:

$x \neq a,$	$y \neq b$	
$x \geq a,$	$b < y \leq c$	algebraic
$[a, \infty)$	$(b, c]$	interval
$\{x \mid a \leq x\}$	$\{y \mid b < y \leq c\}$	Set

Odd & Even Fns: Given $y = f(x)$
Then f is even if $f(-x) = f(x)$
or f is odd if $f(-x) = -f(x)$
otherwise it is just neither.

Piecewise Fns:

$$y = f(x) = \begin{cases} 1, & x < -1 \\ -x, & -1 \leq x \leq 0 \\ x^2, & x > 0 \end{cases}$$

Composite fns: we put a value in one fn and take the value coming out and put it in another fn. This is common in business.

$f(g(x)) = (f \circ g)(x)$
f of g of x

Generally: $f(g(x)) \neq g(f(x))$

Exponential Functions - looks like powers. However, there is a major difference; the variable is in exponent rather than the base and the base must be a positive constant.
The domain for exp fns is all reals.

Properties: $\lim_{x \to \infty} a^x = \infty$ $\quad \lim_{x \to \infty} a^{-x} = 0$ $\quad a^0 = 1$

Rules: (if $a > 0$ & $b > 0$ & $x \in \mathbb{R}$ & $y \in \mathbb{R}$)

1. $a^x \cdot a^y = a^{x+y}$
2. $\frac{a^x}{a^y} = a^{x-y}$
3. $(a^x)^y = a^{xy}$
4. $(ab)^x = a^x b^x$
5. $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

Do these rules look familiar?

Inverse Fns: A fn needs to be injective to be invertible. The symbol f^{-1} is read "f inverse" It does not mean f to the power of -1 or reciprocal. For the reciprocal, use $[f(x)]^{-1}$.

Testing for Inverse Fns: If $(f \circ g)(x) = (g \circ f)(x) = x$ then f and g are inverses of each other, otherwise they are not.

Finding Inverses: Set $y = f(x)$; Solve for x; swap x & y; $f^{-1}(x) = y$

Logarithmic Fns: We can see that exponential fns are one-to-one, so it must have an inverse. It is called the logarithmic fn.

$y = \log_a x$ is the inverse of $y = a^x$ when $a > 0$, $a \neq 1$.

By convention: $\log x = \log_{10} x$ - common log
$\ln x = \log_e x$ - natural log

Properties of Logarithms.

Base a: $a^{\log_a x} = x$, $\log_a a^x = x$, $a > 1$, $x > 0$

Base e: $e^{\ln x} = x$, $\ln e^x = x$, $x > 0$

Product: $\log_a xy = \log_a x + \log_a y$

Quotient: $\log_a x/y = \log_a x - \log_a y$

→ Power: $\log_a x^y = y \log_a x$

Change Base: $\log_a x = \ln x / \ln a$

Most useful: $\ln(a^x a^y) = (x+y)\ln a$, $\ln\left(\frac{a^x}{a^y}\right) = (x-y)\ln a$

Trigonometric Fns: radians is an angle measure as a ratio of the arc to the radius.

Syr	$\sin\theta = y/r$	$\csc\theta = 1/\sin\theta = r/y$
Cxr	$\cos\theta = x/r$	$\sec\theta = 1/\cos\theta = r/x$
tyx	$\tan\theta = y/x$	$\cot\theta = 1/\tan\theta = x/y$

Period is the smallest value p such that $f(\theta+p) = f(\theta)$.

Special Angles:

θ	$\theta°$	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	0	1	0
$\pi/6$	30	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
$\pi/4$	45	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$\pi/3$	60	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$\pi/2$	90		0	undef

Notation

- $\forall$ - for all
- IFF - if and only if
- $\Leftrightarrow$ - if and only if
- DNE - does not exist
- $\exists$ - there exists
- $\in$ - element of
- $|x|$ - absolute value
- $\{\ \}$ - set
- $(\ )$ - open interval
- $[\ ]$ - closed interval
- $\therefore$ - therefore
- s.t. - such that

- α - alpha
- β - beta
- δ - delta
- ε - epsilon
- θ - theta
- $\mathbb{R}$ - all reals
- $\mathbb{Q}$ - set of rationals
- $\mathbb{Z}$ - set of integers
- $\mathbb{N}$ - $1, 2, 3, 4 \ldots$
- $\mathbb{N}_0$ - $0, 1, 2, 3, \ldots$