Calc 12 – Chp 2 Review / Ref Sheet

Properties of Limits:

$\lim_{x\to c} k = k$ and $\lim_{x\to c} x = c$

Limit Theorems: Given $L, M, c, k \in \mathbb{R}$ and

$\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = M$ ← finite

Sum Rule: $\lim_{x\to c} [f(x) + g(x)] = L + M$

Difference Rule: $\lim_{x\to c} [f(x) - g(x)] = L - M$

Product Rule: $\lim_{x\to c} [f(x) \cdot g(x)] = L \cdot M$

Constant Multiplier Rule: $\lim_{x\to c} k \cdot f(x) = k \cdot L$

Quotient Rule: $\lim_{x\to c} [f(x)/g(x)] = L/M$, $M \neq 0$

Power Rule: $r, s \in \mathbb{Z}$: $\lim_{x\to c} [f(x)]^{r/s} = L^{r/s}$

tangent slope $= \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

One-sided Limits:

left-hand: $\lim_{x\to c^-} f(x) = L$ right-hand: $\lim_{x\to c^+} f(x) = L$

Two-sided Limit:

$\lim_{x\to c} f(x) = L \iff \lim_{x\to c^+} f(x) = L$ and $\lim_{x\to c^-} f(x) = L$

Horizontal Asymptotes:

The line $y = L$ is a H.A. of the curve $f(x)$ if either

$\lim_{x\to -\infty} f(x) = L$ or $\lim_{x\to\infty} f(x) = L$

Theorem: If $r > 0$ is a rational number ($\mathbb{Q}$), then

$\lim_{x\to\infty} \dfrac{1}{x^r} = 0.$

Theorem: If $r > 0$ is a $\mathbb{Q}$ such that (s.t.) x^r is defined for all ($\forall$) x, then

$$\lim_{x \to -\infty} \frac{1}{x^r} = 0$$

Vertical Asymptotes:

The line $x = c$ is a V.A. of the curve $f(x)$ if either

$$\lim_{x \to c^-} f(x) = \pm\infty \quad \text{or} \quad \lim_{x \to c^+} f(x) = \pm\infty$$

Finding End Behaviour Models

Definitions:

Left-end behaviour model for 'f' $\Leftrightarrow \lim_{x \to -\infty} \frac{f(x)}{g(x)} = 1$

Right-end behaviour model for 'f' $\Leftrightarrow \lim_{x \to \infty} \frac{f(x)}{g(x)} = 1$

Continuity - Big idea is that if we can draw a fn without lifting our pen, then the fn is continuous. However in math we need to show this with limits and theorems.

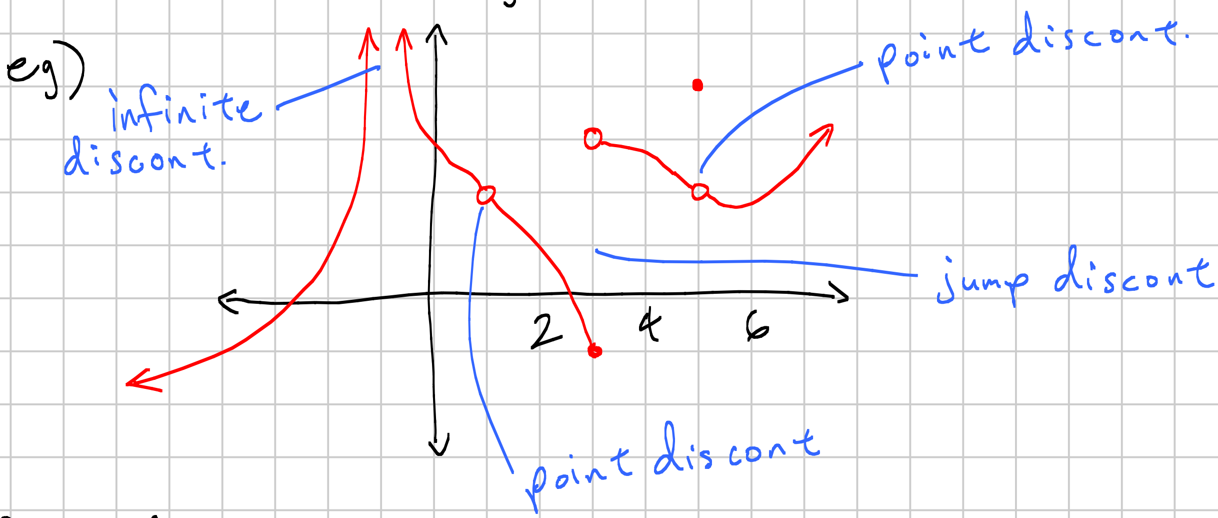
Definition: A fn 'f' is continuous at an interior point 'a' of its domain if

$$\lim_{x \to a} f(x) = f(a)$$

If 'f' is not continuous at 'a', then we say 'f' is discontinuous at 'a' and 'a' is a point of discontinuity.

Types of Discontinuity

eg)



Point discontinuity is removable.

Definition: A fn 'f' is continuous on an interval if and only if (iff) it is continuous at every point in the interval.

Definition: A continuous fn 'f' is one that is continuous at every point in its domain.

Theorem: Any polynomial is cont. on $\mathbb{R}$.

Theorem: Any rational fn is cont on its domain, ie. anywhere except at NPV's.

Theorem: The following fns are cont. in their domains: roots, trig, inverse trig, exponentials, logarithms.

Theorem: Composition of Cont. Fns. If 'g' is cont. at 'a' and 'f' is cont at $g(a)$, then '$f \circ g$' is cont. at 'a'.

Intermediate Value Theorem (IVT): Suppose that 'f' is cont. on an interval $[a, b]$ and let N be any number between $f(a)$ & $f(b)$ where $f(a) \neq f(b)$. Then there exists a number $c \in (a, b)$ s.t. $f(c) = N$.

Average Rate of Change: $\dfrac{\Delta y}{\Delta x}$

Slope of a Curve at $y = f(x)$ at the point $P(a, f(a))$ is the number:

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Use point-slope form: $y - f(a) = m(x - a)$

Sandwich (Squeeze) Theorem

If $f(x) \leq g(x) \leq h(x)$ when 'x' is near 'a' and

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$$

then $\lim_{x \to a} g(x) = L$.

Show that $\sqrt{2}$ is between 1 & 2 using IVT.

$x = \sqrt{2}$

$x^2 = 2$

$x^2 - 2 = 0$

Let $f(x) = x^2 - 2$

We are looking for a 'c' between 1 & 2 s.t. $f(c) = 0$

$\therefore$ $a = 1$, $b = 2$, & $N = 0$ in IVT.

$f(1) = 1^2 - 2 = -1 < 0$ N

$f(2) = 2^2 - 2 = 2 > 0$

Thus $f(1) < 0 < f(2)$

'f' is a polynomial, $\therefore$ 'f' is continuous.

So, by IVT, $\exists c \in (1, 2)$ s.t. $f(c) = 0$

Study Tip! Review notes, but also practice questions. Find review questions or even redo some HW Q's.

Stewart: Chp 2.2, 2.3, 2.5, 2.6, 2.7