Order of Operations - BEDMAS Review

Note Title 2015-12-20

		sometimes
B - Brackets		P - Parentheses
E - Exponents		E
D - Division	} left to right	M
M - Multiplication	}	D
A - Addition	} left to right	A
S - Subtraction	}	S

All brackets are done first; this includes functions such as absolute value and square root. Inner brackets are also done first when brackes are nested.

eg) $-7+3\div 2\times\sqrt{(-7+5)^2+12}\div(-9)-5+19$

Inner brackets first:

Then exponents:

even # of -'ve signs is positive

odd # of -'ve signs is negative

Now:

Brackets (functions) next:

Now:

Another way to view:

(after brackets are done)

- multiplies in numerator
- divides in denominator
- adds to the left
- subtracts to the right.

Divide and Multiply left to right:

Now:

Add and Subtract left to right:

Continuing from another view:

Same answer.

Reminder: If we ignore order of operations, we will get varying values.

eg) $2 + 3 \times 4$

incorrect correct

Brackets around a negative number is not part of order of operations; it simply makes the negative number explicit.

eg) $3 + (-2)^4$ vs $3 - (2)^4$

Alternate symbols: Multiply: $*$, $\cdot$, $*$, $()()$ ← no operator

Divide: $\div$, $/$, $\frac{()}{()}$

Exercises

Evaluate:

1) $2^3 \cdot 3^4 \div 9 \times 7 \div 2$

2) $(2+3)^2 + (3+4)^2 \div 7 - 15 \div 5$

3) $4^0 \div 3 + 7/9 - 3$

4) $(36 \div 2^2 + 11)^3 - 4^5$

5) $5 \times 4^2 - (2^3 + 3^3) \div 5$

6) $(5 - (-4))^3 - (21 \div 7)^4$

7) $-2^4 \cdot 5 + 16 \div (-2)^3$

8) $\dfrac{2^4 (4^3 \div 2^2) - 4^0}{3(3^4 + 2^2)}$

9) $(-2)^4 - (-3)^3 \div (-9)^0 \times 2^3$

10) $\left((-12)^3 \div (-6)^4\right)^2 \cdot 8 + (5)^3 \div 6 - 3/8$

Answers: 1) 252 2) 29 3) $-17/9$ 4) 6976 5) 73 6) 648
7) -82 8) 1 9) 232 10) $249/72$

Polynomials and Number Representation Review

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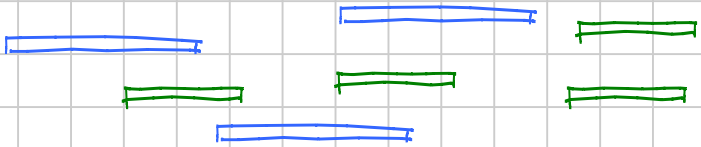
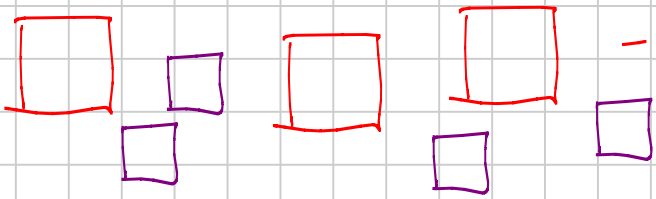
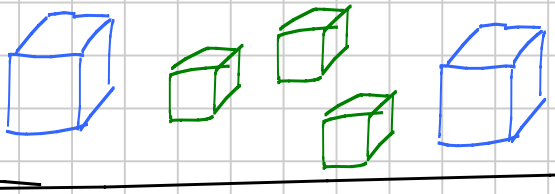
Analogy - this is a way to think of the polynomials; not what it actually means.

Constants: - countable objects.

Linear Terms:  - line objects

Quadratic Terms:  - square objects

Cubic Terms:  - cube objects

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Adding & Subtracting Polynomials - only add and subtract like terms; because cubes and squares are different. BEDMAS still applies!

eg) Simplify:

$$5 - 7x^2 + 3x - 4x^3 + 8x^2 + 6x - 13 - 5x^3 + 20 - 9x$$

Multiplying Polynomials - Remember to distribute (FOIL for 2 binomials) and Power Laws!

$$(a+b)(c+d) = ac + ad + bc + bd$$

$$x^m \cdot x^n = x^{m+n} \qquad \frac{x^m}{x^n} = x^{m-n}$$

eg) $\frac{3x^3}{6x^2}$ =

$3x^2(-4x)(5x)$ =

$(x-3)(2x+2)(-3x-2)$

$(-3x-4)(-5x+6)$

Dividing Polynomials – factor numerator and denominator before cancelling. Must fully factor otherwise cancelling is NOT allowed. Also, when cancelling, the factor becomes 1 not 0 or nothing!

eg) $\frac{4+2x}{4x+4}$

$\frac{x^2+3x+2}{x^2-2x-3}$

$\frac{2x^2+10x+8}{2x^2+2x-24}$

$(x^2-3x-18) \div (x+3)$

$(x+6) \div (x^2+4x-12)$

Last Tip: When simplifying, if there is a rational, convert to a single rational by using a common denominator.

Simplify:

1) $\frac{8x^2+4xy+6xy^2}{2x^2y}$

2) $\dfrac{9x^2y + 6x^2y^2 - 12xy^2}{3x^2y^3}$

3) $3x^2 - 4y - x^2 + 7y + xy - 2xy$

4) $(3x+2y)^2 - (2x-4y)^2 + 3xy - 6y^2$

5) $x - 5 + (x^2 + 6x + 9) \div (x^2 + 2x - 3)$

6) $\dfrac{2y^2+7y+6}{y+2} - \dfrac{y^2+2y-15}{y-3}$

7) $\dfrac{k^2+3k-4}{k+4} - \dfrac{k-2}{k^2+k-6}$

8) $\dfrac{x^2+x-6}{x^2+2x-3} \cdot \dfrac{x^2+3x-4}{x+4}$

9) $\dfrac{x^2-3x-10}{x^2-4x-5} \cdot \dfrac{x+1}{x^2+6x+8}$

Answers: 1) $\dfrac{4x+2y+3y^2}{xy}$ 2) $\dfrac{3x+2xy-4y}{xy^2}$

3) $2x^2 - xy + 3y$ 4) $5x^2 + 31xy - 18y^2$ 5) $\dfrac{x^2-5x+8}{x-1}$

6) $y-2$ 7) $\dfrac{k^2+2k-4}{k+3}$ 8) $x-2$ 9) $\dfrac{1}{x+4}$

Algebra Review

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Algebra is a set of rules for manipulating expressions and equations. An important concept is that the rules apply whether we are manipulating numbers, variables or functions. You already know most of the rules because they follow the order of operations, combining only 'like' terms in polynomials, and power laws.

There are shortcuts, but in general, we want to do BEDMAS in reverse, SAMDEB. However it is easier for most students to expand any brackets, collect like terms, move all variables to one side, then solve.

eg) $3(x-2) - 2(x+5) = -4(x-6)$

expand
collect like terms
do inverse add/sub to both sides

simplify
do inverse mult to both sides

simplify

Inverse operations:

Add	<=>	Subtract
Multiply	<=>	Divide
x^2	<=>	$\sqrt{x}$
x^3	<=>	$\sqrt[3]{x}$
$\sin(x)$	<=>	$\sin^{-1}(x)$
$\cos(x)$	<=>	$\cos^{-1}(x)$
$\tan(x)$	<=>	$\tan^{-1}(x)$
e^x	<=>	$\ln(x)$
10^x	<=>	$\log(x)$
$\int f(x)dx$	<=>	$\frac{d}{dx} f(x)$
substitute	<=>	subst back

eg) $\frac{1}{5}(x+20) - \frac{1}{3}(2x-9) = -\frac{1}{2}(5x+8)$

expand
do inverse add/sub
use common denom
collect like terms
do inverse mult
simplify

An important concept in algebra is substitution. This can be used whenever you see the same expression; you simply substitute the corresponding expression. By using substitution it is possible to eliminate extra variables and we are left with one equation and a single variable. Although you learn elimination and substitution in Math 10, you will use substitution almost exclusively when doing math other than system of linear equations.

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eg) convert 5 km into miles

eg) Solve $5x + 2y = 7$ & $y = 3x - 2$

Substitute
expand
do inverse subtract
simplify
do inverse mult
simplify
substitute
simplify

An important reminder for substitution is to put brackets around the substituting expression to prevent any sign errors. If there are 2 variables, you must solve for both variables.

Your 3 main tools for solving any problem are: substitution, factorization, and algebra.

In PreCalc 12, you will need to use special algebraic methods from PreCalc 11 such as the quadratic formula, complete the square, and solving radical equations. One important identity is

$$\sqrt{x^2} = |x| \quad \text{or} \quad \sqrt{x^2} = \pm x$$

A better understanding of $|x|$ can be seen using the piecewise function: $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

eg) Find the domain of $\sqrt{4-x^2}$

Solve:

1) $3(x-4) + 2(2x+1) = -4(2x-5) + 2(3x+4)$

2) $3x + 4y = 14 + 2y - x$ & $y = 2x + 3$

3) $-3x + 2y = 21 + 3x - y$ & $2y = -4x + 6$

4) $2x^2 + 2(x-1) = x^2 - 3(x+2)$

5) $(2x+3)(x-5) = (x+3)(x-2)$

6) $x^2 + (y+2)^2 = -6 + 5x - y$ $\qquad y = 2x - 5$

7) $2\sqrt{x^2-2x+3} = 3\sqrt{2x-2}$

8) Find domain $\sqrt{x^2+5x-14}$

Answers: 1) $x = 38/9$ 2) $x=1,\ y=5$ 3) $x=-1,\ y=5$
4) $x=-4$ or $x=-1$ 5) $x=-1$ or $x=9$
6) $x=2,\ y=-1$ or $x=1,\ y=-3$ 7) $x=3/2$ or $x=5$
8) D: $x \le -7$ or $x \ge 2$

Word Problems Review

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Common Formulas:

Perimeter of rectangle: $2(l+w)$ or $2l+2w$
" triangle: $a+b+c$
Circumference of circle: $2\pi r$
Diameter: $2r$
Area of square: s^2
" rectangle: lw
" circle: πr^2
" triangle: $\frac{1}{2}bh$
Surface Area of cube: $6s^2$
" rectangular prism: $2(lw+lh+wh)$
" sphere: $4\pi r^2$
" cylinder: $2\pi r^2 + 2\pi rh$ or $2\pi r(r+h)$
" square pyramid: $b^2 + 2b\sqrt{(b/2)^2+h^2}$
" cone: $\pi r^2 + \pi r\sqrt{r^2+h^2}$
Volume of cube: s^3
" rectangular prism: lwh
" cylinder: $\pi r^2 h$
" cone: $\pi r^2 h/3$
" square pyramid: $b^2 h/3$
" sphere: $\frac{4}{3}\pi r^3$

Revenue = price (# of units)
Profit = Revenue - Cost
Profit Margin = Profit (100%) / Revenue
Equity = Assets - Liability
Simple Interest - $A = A_0(1+i)t$
Compound Interest - $A = A_0\left(1+\frac{i}{n}\right)^{nt}$ n - compound/yr
FV (Savings) = $PMT \cdot \frac{\left[\left(1+\frac{i}{n}\right)^{nt}-1\right]}{i/n}$

PV (loans) = $PMT \cdot \frac{\left[1-\left(1+\frac{i}{n}\right)^{-nt}\right]}{i/n}$

Half-life = $N = N_0 e^{-\lambda t}$ $\lambda = \frac{\ln 2}{\text{half-life}}$
or = $N = N_0\left(\frac{1}{2}\right)^{t/\text{half-life}}$

Substitution is frequently used for word problems to create solvable equations.

eg) A box has a length that is triple the width, and a height that is double the width. The surface area is 864 cm^2. The manufacturer wants to make the surface area 1152 by increasing each dimension by the same amount. Find the old and new dimensions.

from problem

standard formula

substitute $l=3w$ & $h=2w$

expand

collect like terms

substitute $SA=864$

algebra & reduce

Original dimensions

Find zero on calculator

Verify: $2(lw+lh+wh)$

eg) A can has a volume of 750 mL. It's diameter is half the height. The manufacturer wants to increase the volume to 2 L and have the diameter and height increase by the same dimension. Find the initial radius and the dimensional increase.

start with volume of cylinder
formula for diameter
formula for height from problem.
substitute $d = 2r$
simplify
substitute $h = 4r$ & $V = 750$
simplify
algebra

$1\ mL = 1\ cm^3$
from word problem
substitute $d = 2r$ & $h = 4r$
substitute $r = 3.91$

substitute $V = 2000$

find zero on calculator

Strategy: Draw diagrams, write formulas, look for substitutions, re-read question, then solve!

Exercises: Solve:

1) The sum of triple the first number and double the second number is 3. The difference of double the first and triple the second is 28. Find the two numbers.

2) There are 15 dimes and quarters worth $2.55. How many of each are there?

3) Small buses hold 18 students and big ones hold 32 students. 9 buses are used to take 218 students for a field trip. How many of each bus are used?

4) The product of double the first number and triple of a second is 144. The sum of 5 times the first and double the second is 32.

5) A box has 500 cm^2 of surface area. Its length and width are twice the height. The area needs to be increased to 800 cm^2 by increasing the length and height by the same amount. Find the original and new dimensions.

Use Calculator to Solve for Zero or root.

Answers: 1) 5 and -6 2) 8 dimes, 7 quarters 3) 5 small, 4 big 4) ($12/5$ and 10) or (4 and 6) 5) ($h = 5.6$, $l = w = 11.2$) and ($h = 9.1$, $l = 14.7$, $w = 11.2$)