PreCalc 11 Final Review Chp 3

Note Title 2016-05-25

3.1) Factoring Polynomial Expressions.

- factoring allows us to determine smaller values that form a desired product. eg) $3 \cdot 4 = 12$, $x^2+3x+2 = (x+2)(x+1)$
- If not obvious, use PFS (product, factor, sum) method

ax^2+bx+c : product $= a \cdot c$ $\quad F = f_1 \cdot f_2 = P \quad S = f_1 + f_2 = b$

eg) $3x^2 - 64x - 44 \qquad P: 3(-44) = -132$

f_1	f_2		S
-132	$+1$	$=$	-131
-66	$+2$	$=$	-64 ✓

$3x^2 - 66x + 2x - 44$ add x's

$3x(x-22) + 2(x-22)$ factor pairs.

$= (3x+2)(x-22)$

Factoring with Rational Coefficients

eg) $x^2 - 1.6x - .8$

$= x^2 - \frac{16}{10}x - \frac{8}{10}$

$= x^2 - \frac{8}{5}x - \frac{4}{5}$

$= \frac{1}{5}(5x^2 - 8x - 4)$

$= \frac{1}{5}(5x+2)(x-2)$

Factoring with Patterns

eg) $4(2x-3)^2 + 5(2x-3) - 6$ $\qquad y = 2x-3$

$= 4y^2 + 5y - 6$

$= (4y-3)(y+2)$

$= (4(2x-3)-3)((2x-3)+2)$

$= (8x-15)(2x-1)$

* Difference of Squares. $\quad (a+b)(a-b) = a^2 - b^2$

eg) $(x-1)^2 - 4(y+2)^2 = [(x-1) - 2(y+2)][(x-1) + 2(y+2)]$

$= [x - 2y - 5][x + 2y + 3]$

Checking for factors:

eg) Is $2x-1$ a factor of $6x^2 - 17x + 12$?

$(2x-1)(ax+b) = 2ax^2 + 2bx - ax - b$

$2a = 6 \qquad -b = 12 \qquad 2b - a = -17$

$a = 3 \qquad b = -12 \qquad 2(-12) - 3 = -17$

$-27 \neq -17$, so $2x-1$ is not a factor.

3.2) Same as before but add "=0"

Property: $a \cdot 0 = 0$ $\quad 0 \cdot a = 0$

So factor=0 is a solution.

eg) $(2x-5)(3x+1) = 0$

$2x-5=0$ $\quad$ $3x+1=0$

$2x=5$ $\quad$ $3x=-1$

$x=5/2$ $\quad$ $x=-1/3$

Factoring out constants doesn't change solutions.

eg) $7x^2+35x-42=0$ $\quad y_1$

$7(x^2+5x-6)=0$

$x^2+5x-6=0$ $\quad y_2$

Square radical equations to solve. Check for extraneous

eg) $\sqrt{x} = -x+2$

$\sqrt{x}^2 = (-x+2)^2$

$x = x^2-4x+4$

$0 = x^2-5x+4$

$0 = (x-4)(x-1)$

$x=4$

$\sqrt{4} = -4+2$

$2=-2$

extraneous

$x=1$

$\sqrt{1} = -1+2$

$1=1$ ✓

3.3) Complete the Square $\quad y = ax^2+bx+c$

1 - Divide 'x' terms by 'a'

2 - Add new $(b/2)^2$ & sub new $a(b/2)^2$

3 - $y = a(x-b/2)^2 - a(b/2)^2 + c$

original b

new b

eg) $y = -3x^2+12x+5$

$= -3(x^2-4x)+5$ $\quad$ 1

$= -3(x^2-4x+4)-(-12)+5$ $\quad$ 2

$= -3(x-2)^2+17$ $\quad$ 3

3.4) Quadratic Formula (QF): $\quad ax^2+bx+c=0$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

3.5) Discriminant: $b^2-4ac = \Delta$

$b^2-4ac>0$, 2 real roots - Δ not perfect square, irrational roots

- Δ is perfect square, rational roots

↳ 1, 4, 9, 16, 25, 36, 49, ...

$b^2-4ac=0$, 1 real root

$b^2-4ac<0$, no real roots, not no solutions.

Determine k from nature of roots.

eg) Determine k so there are no real roots.

$$2x^2 - 5x + k = 0$$

$$b^2 - 4ac < 0$$

$$25 - 4(2)(k) < 0$$

$$-8k < -25$$

$$k > 25/8$$