

PreCalc 12 Chapter 4 Review 2017 v1

Answer Section

MULTIPLE CHOICE

1. ANS: C PTS: 0 DIF: Easy REF: 4.1 Combining Functions Graphically
LOC: 12.RF1 TOP: Relations and Functions
KEY: Conceptual Understanding | Procedural Knowledge
2. ANS: C PTS: 0 DIF: Easy REF: 4.2 Combining Functions Algebraically
LOC: 12.RF1 TOP: Relations and Functions
KEY: Conceptual Understanding | Procedural Knowledge
3. ANS: D PTS: 0 DIF: Easy REF: 4.2 Combining Functions Algebraically
LOC: 12.RF1 TOP: Relations and Functions
KEY: Conceptual Understanding | Procedural Knowledge
4. ANS: D PTS: 0 DIF: Easy REF: 4.2 Combining Functions Algebraically
LOC: 12.RF1 TOP: Relations and Functions
KEY: Conceptual Understanding | Procedural Knowledge
5. ANS: A PTS: 0 DIF: Easy REF: 4.2 Combining Functions Algebraically
LOC: 12.RF1 TOP: Relations and Functions
KEY: Conceptual Understanding | Procedural Knowledge
6. ANS: D PTS: 0 DIF: Easy REF: 4.3 Introduction to Composite Functions
LOC: 12.RF1 TOP: Relations and Functions
KEY: Conceptual Understanding | Procedural Knowledge
7. ANS: B PTS: 0 DIF: Easy REF: 4.2 Combining Functions Algebraically
LOC: 12.RF1 TOP: Relations and Functions
KEY: Conceptual Understanding | Procedural Knowledge
8. ANS: C PTS: 0 DIF: Easy REF: 4.2 Combining Functions Algebraically
LOC: 12.RF1 TOP: Relations and Functions
KEY: Conceptual Understanding | Procedural Knowledge
9. ANS: D
Do $f(-1)$ first.

PTS: 0 DIF: Easy REF: 4.3 Introduction to Composite Functions
LOC: 12.RF1 TOP: Relations and Functions
KEY: Conceptual Understanding | Procedural Knowledge
10. ANS: C PTS: 0 DIF: Easy REF: 4.3 Introduction to Composite Functions
LOC: 12.RF1 TOP: Relations and Functions
KEY: Conceptual Understanding | Procedural Knowledge
11. ANS: C PTS: 0 DIF: Easy
REF: 4.4 Determining Restrictions on Composite Functions LOC: 12.RF1
TOP: Relations and Functions KEY: Conceptual Understanding | Procedural Knowledge
12. ANS: D PTS: 0 DIF: Easy
REF: 4.4 Determining Restrictions on Composite Functions LOC: 12.RF1
TOP: Relations and Functions KEY: Conceptual Understanding | Procedural Knowledge
13. ANS: D PTS: 0 DIF: Easy
REF: 4.4 Determining Restrictions on Composite Functions LOC: 12.RF1
TOP: Relations and Functions KEY: Conceptual Understanding | Procedural Knowledge

14. ANS: C PTS: 0 DIF: Moderate REF: 4.1 Combining Functions Graphically
 LOC: 12.RF1 TOP: Relations and Functions
 KEY: Conceptual Understanding | Procedural Knowledge
15. ANS: B PTS: 0 DIF: Moderate REF: 4.1 Combining Functions Graphically
 LOC: 12.RF1 TOP: Relations and Functions
 KEY: Conceptual Understanding | Procedural Knowledge
16. ANS: B PTS: 0 DIF: Moderate REF: 4.2 Combining Functions Algebraically
 LOC: 12.RF1 TOP: Relations and Functions
 KEY: Conceptual Understanding | Procedural Knowledge
17. ANS: B PTS: 0 DIF: Moderate REF: 4.3 Introduction to Composite Functions
 LOC: 12.RF1 TOP: Relations and Functions
 KEY: Conceptual Understanding | Procedural Knowledge
18. ANS: D
 Expand $h(x)$ then complete the square.
- PTS: 0 DIF: Moderate REF: 4.4 Determining Restrictions on Composite Functions
 LOC: 12.RF1 TOP: Relations and Functions
 KEY: Conceptual Understanding | Procedural Knowledge
19. ANS: A PTS: 0 DIF: Moderate REF: 4.1 Combining Functions Graphically
 LOC: 12.RF1 TOP: Relations and Functions
 KEY: Conceptual Understanding | Procedural Knowledge
20. ANS: D PTS: 0 DIF: Moderate REF: 4.3 Introduction to Composite Functions
 LOC: 12.RF1 TOP: Relations and Functions
 KEY: Conceptual Understanding | Procedural Knowledge
21. ANS: D PTS: 0 DIF: Moderate
 REF: 4.4 Determining Restrictions on Composite Functions LOC: 12.RF1
 TOP: Relations and Functions KEY: Conceptual Understanding | Procedural Knowledge
22. ANS: B PTS: 0 DIF: Moderate
 REF: 4.4 Determining Restrictions on Composite Functions LOC: 12.RF1
 TOP: Relations and Functions KEY: Conceptual Understanding | Procedural Knowledge
23. ANS: C PTS: 0 DIF: Moderate REF: 4.3 Introduction to Composite Functions
 LOC: 12.RF1 TOP: Relations and Functions
 KEY: Conceptual Understanding | Procedural Knowledge
24. ANS: C PTS: 0 DIF: Moderate
 REF: 4.4 Determining Restrictions on Composite Functions LOC: 12.RF1
 TOP: Relations and Functions KEY: Conceptual Understanding | Procedural Knowledge

SHORT ANSWER

25. ANS:
 a) $g(f(-1)) = 24$
 b) $g(g(-1)) = -1$

PTS: 0 DIF: Easy REF: 4.3 Introduction to Composite Functions
 LOC: 12.RF1 TOP: Relations and Functions
 KEY: Conceptual Understanding | Procedural Knowledge

26. ANS:

$$p(x) = -15x^2 + 3x + 12$$

PTS: 0 DIF: Easy REF: 4.2 Combining Functions Algebraically

LOC: 12.RF1 TOP: Relations and Functions

KEY: Conceptual Understanding | Procedural Knowledge

27. ANS:

$$f(g(x)) = 3\sqrt{x+6} + 6$$

Domain: $x \geq -6$

PTS: 0 DIF: Easy REF: 4.4 Determining Restrictions on Composite Functions

LOC: 12.RF1 TOP: Relations and Functions

KEY: Conceptual Understanding | Procedural Knowledge

28. ANS:

$$g(f(x)) = \sqrt{6x+8}$$

Domain: $x \geq -\frac{4}{3}$

PTS: 0 DIF: Easy REF: 4.4 Determining Restrictions on Composite Functions

LOC: 12.RF1 TOP: Relations and Functions

KEY: Conceptual Understanding | Procedural Knowledge

29. ANS:

Forms of the equation may vary.

$$g(f(x)) = 5x + 8\sqrt{x}$$

Domain: $x \geq 0$

PTS: 0 DIF: Easy REF: 4.4 Determining Restrictions on Composite Functions

LOC: 12.RF1 TOP: Relations and Functions

KEY: Conceptual Understanding | Procedural Knowledge

30. ANS:

$$d(x) = \sqrt{3-x} - \sqrt{x+2}$$

Domain: $-2 \leq x \leq 3$

PTS: 0 DIF: Moderate REF: 4.2 Combining Functions Algebraically

LOC: 12.RF1 TOP: Relations and Functions

KEY: Conceptual Understanding | Procedural Knowledge

31. ANS:

Possible functions are:

$$p(x) = \sqrt{x+2}, q(x) = x, \text{ and } r(x) = \frac{1}{x}$$

PTS: 0 DIF: Moderate REF: 4.4 Determining Restrictions on Composite Functions

LOC: 12.RF1 TOP: Relations and Functions

KEY: Conceptual Understanding | Procedural Knowledge | Problem-Solving Skills

32. ANS:

$$q(x) = \frac{2-x^2}{\sqrt{x+3}}$$

Domain: $x > -3$

PTS: 0 DIF: Moderate REF: 4.4 Determining Restrictions on Composite Functions

LOC: 12.RF1 TOP: Relations and Functions

KEY: Conceptual Understanding | Procedural Knowledge

33. ANS:

a) $f(g(2)) = 90$

b) $g(f(-3)) = 8$

PTS: 0 DIF: Moderate REF: 4.3 Introduction to Composite Functions

LOC: 12.RF1 TOP: Relations and Functions

KEY: Conceptual Understanding | Procedural Knowledge

34. ANS:

Since $f(g(x)) = g(f(x)) = x$, the functions are inverses.

PTS: 0 DIF: Moderate REF: 4.3 Introduction to Composite Functions

LOC: 12.RF1 TOP: Relations and Functions

KEY: Conceptual Understanding | Procedural Knowledge

35. ANS:

a) $h(g(f(2))) = 2$

b) $f(g(h(2))) = 14 - 8\sqrt{2}$

PTS: 0 DIF: Moderate REF: 4.3 Introduction to Composite Functions

LOC: 12.RF1 TOP: Relations and Functions

KEY: Conceptual Understanding | Procedural Knowledge

36. ANS:

$$f(g(x)) = \frac{x^2 + 1}{2}$$

Domain: $x \in \mathbb{R}$; Range: $y \geq 0.5$

PTS: 0 DIF: Moderate REF: 4.3 Introduction to Composite Functions

LOC: 12.RF1 TOP: Relations and Functions

KEY: Conceptual Understanding | Procedural Knowledge

PROBLEM

37. ANS:

a) $f(g(x)) = f(x^3 - 3)$

$$f(g(x)) = 2(x^3 - 3) + 1$$

$$f(g(x)) = 2x^3 - 6 + 1$$

$$f(g(x)) = 2x^3 - 5$$

This is a cubic function; its domain is: $x \in \mathbb{R}$; its range is $y \in \mathbb{R}$.

b) $g(f(x)) = g(2x + 1)$

$$g(f(x)) = (2x + 1)^3 - 3$$

$$g(f(x)) = (2x + 1)(4x^2 + 4x + 1) - 3$$

$$g(f(x)) = 8x^3 + 12x^2 + 6x + 1 - 3$$

$$g(f(x)) = 8x^3 + 12x^2 + 6x - 2$$

This is a cubic function; its domain is: $x \in \mathbb{R}$; its range is $y \in \mathbb{R}$.

PTS: 0 DIF: Moderate REF: 4.3 Introduction to Composite Functions

LOC: 12.RF1 TOP: Relations and Functions

KEY: Conceptual Understanding | Procedural Knowledge | Communication

38. ANS:

$$\begin{aligned} \text{a) } f(g(x)) &= f(2x^2 - 3x) \\ f(g(x)) &= -(2x^2 - 3x) + 2 \\ f(g(x)) &= -2x^2 + 3x + 2 \end{aligned}$$

This is a quadratic function; its domain is: $x \in \mathbb{R}$

Use graphing technology to determine the range.

From the graph, the range is: $y \leq 3.125$

$$\begin{aligned} \text{b) } g(f(x)) &= g(-x + 2) \\ g(f(x)) &= 2(-x + 2)^2 - 3(-x + 2) \\ g(f(x)) &= 2x^2 - 8x + 8 + 3x - 6 \\ g(f(x)) &= 2x^2 - 5x + 2 \end{aligned}$$

This is a quadratic function; its domain is: $x \in \mathbb{R}$

Use graphing technology to determine the range.

From the graph, the range is: $y \geq -1.125$

$$\begin{aligned} \text{c) } f(f(x)) &= f(-x + 2) \\ f(f(x)) &= -(-x + 2) + 2 \\ f(f(x)) &= x - 2 + 2 \\ f(f(x)) &= x \end{aligned}$$

This is a linear function; its domain is: $x \in \mathbb{R}$; and its range is: $y \in \mathbb{R}$

$$\begin{aligned} \text{d) } g(g(x)) &= g(2x^2 - 3x) \\ g(g(x)) &= 2(2x^2 - 3x)^2 - 3(2x^2 - 3x) \\ g(g(x)) &= 8x^4 - 24x^3 + 18x^2 - 6x^2 + 9x \\ g(g(x)) &= 8x^4 - 24x^3 + 12x^2 + 9x \end{aligned}$$

This is a quartic function; its domain is: $x \in \mathbb{R}$

Use graphing technology to determine the range.

From the graph, the range is: $y \geq -1.125$

PTS: 0 DIF: Moderate REF: 4.3 Introduction to Composite Functions

LOC: 12.RF1 TOP: Relations and Functions

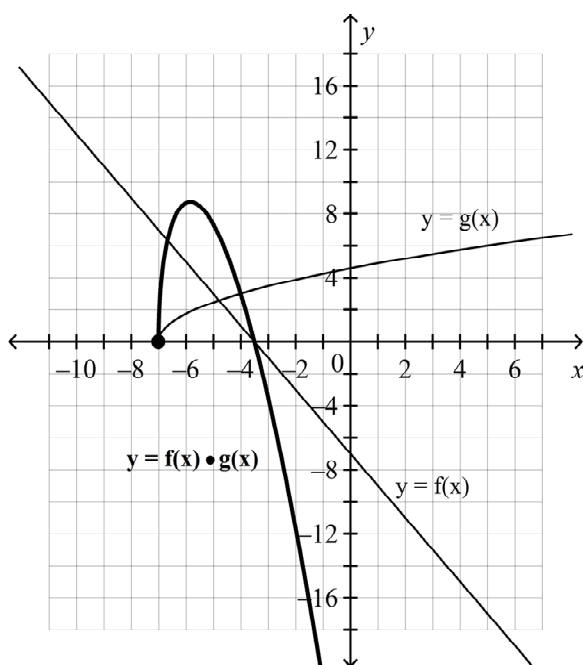
KEY: Conceptual Understanding | Procedural Knowledge | Communication

39. ANS:

- a) From the graph of $y = f(x)$: domain: $x \in \mathbb{R}$; range: $y \in \mathbb{R}$
 b) From the graph of $y = g(x)$: domain: $x \geq -7$; range: $y \geq 0$
 c) From the graphs, approximate values are:

| x | $f(x)$ | $g(x)$ | $f(x) \cdot g(x)$ |
|-----|--------|--------|-------------------|
| -7 | 7 | 0 | 0 |
| -6 | 5 | 1.7 | 8.5 |
| -5 | 3 | 2.4 | 7.2 |
| -4 | 1 | 3.0 | 3.0 |
| -3 | -1 | 3.5 | -3.5 |
| -2 | -3 | 3.9 | -11.7 |

Plot the points at: $(-7, 0)$, $(-6, 8.5)$, $(-5, 7.2)$, $(-4, 3)$, $(-3, -3.5)$, $(-2, -11.7)$
 Join the points with a smooth curve.



- d) The domain of $y = f(x) \cdot g(x)$ is: $x \geq -7$
 The domain is the same as the domain of $y = g(x)$. It is not equal to the domain of $y = f(x)$, which is all real numbers, because the graph of $y = f(x) \cdot g(x)$ does not extend to the left of $x = -7$.

PTS: 0 DIF: Moderate REF: 4.1 Combining Functions Graphically
 LOC: 12.RF1 TOP: Relations and Functions
 KEY: Conceptual Understanding | Procedural Knowledge | Communication

40. ANS:

a) Work backward.

Determine the value of x for which $f(x) = -2$.

From the graph, $x = -1$

Determine the value of x for which $g(x) = -1$.

From the graph, $x = -4$

So, for $f(g(a)) = -2$, $a = -4$

b) Determine the value of x for which $g(x) = 2$

From the graph, $x = 2$

Determine the value of x for which $f(x) = 2$

From the graph, $x = 3$

So, for $g(f(b)) = 2$, $b = 3$

PTS: 0 DIF: Moderate REF: 4.3 Introduction to Composite Functions

LOC: 12.RF1 TOP: Relations and Functions

KEY: Procedural Knowledge | Communication | Problem-Solving Skills

41. ANS:

Determine an explicit equation for $f(g(x))$.

In $f(x) = ax + b$, replace x with $cx + d$.

$$f(g(x)) = a(cx + d) + b$$

$$f(g(x)) = acx + ad + b$$

$$f(g(x)) = acx + (ad + b)$$

Determine an explicit equation for $g(f(x))$.

In $g(x) = cx + d$, replace x with $ax + b$.

$$g(f(x)) = c(ax + b) + d$$

$$g(f(x)) = cax + cb + d$$

$$g(f(x)) = acx + (bc + d)$$

Both composite functions are linear functions.

For example, use the linear functions $f(x) = 2x - 3$ and $g(x) = -x + 5$.

$$f(g(x)) = 2(-x + 5) - 3$$

$$f(g(x)) = -2x + 10 - 3$$

$$f(g(x)) = -2x + 7$$

$$g(f(x)) = -(2x - 3) + 5$$

$$g(f(x)) = -2x + 3 + 5$$

$$g(f(x)) = -2x + 8$$

$f(g(x)) = -2x + 7$ and $g(f(x)) = -2x + 8$ are both linear functions.

PTS: 0 DIF: Difficult REF: 4.3 Introduction to Composite Functions

LOC: 12.RF1 TOP: Relations and Functions

KEY: Conceptual Understanding | Procedural Knowledge | Communication

42. ANS:

Complete the square:

$$\begin{aligned}x^2 + 6x + 5 &= x^2 + 6x + 5 + 4 - 4 \\ &= (x + 3)^2 - 4\end{aligned}$$

$$\text{So, } y = \sqrt{(x + 3)^2 - 4}$$

$$\text{Let } f(g(h(x))) = \sqrt{(x + 3)^2 - 4}.$$

Replace $(x + 3)^2$ with x .

$$\text{Then, } g(h(x)) = (x + 3)^2 \text{ and } f(x) = \sqrt{x - 4}$$

$$\text{Use } g(h(x)) = (x + 3)^2.$$

Replace $x + 3$ with x .

$$\text{Then, } h(x) = x + 3 \text{ and } g(x) = x^2$$

Possible functions are: $f(x) = \sqrt{x - 4}$, $g(x) = x^2$, and $h(x) = x + 3$

PTS: 0 DIF: Difficult REF: 4.4 Determining Restrictions on Composite Functions

LOC: 12.RF1 TOP: Relations and Functions

KEY: Procedural Knowledge | Communication | Problem-Solving Skills