# PreCalc 12 Chapter 5 Review 2017 v1
## Answer Section

### MULTIPLE CHOICE

1. **ANS: D**  
   Use substitution: $2^0$  
   
   | PTS: 1 | DIF: Easy | REF: 5.1 Math Lab: Graphing Exponential Functions | LOC: 12.RF9 | TOP: Relations and Functions | KEY: Procedural Knowledge |

2. **ANS: A**  
   The untransformed range of exponentials is $y > 0$  
   
   | PTS: 1 | DIF: Easy | REF: 5.1 Math Lab: Graphing Exponential Functions | LOC: 12.RF9 | TOP: Relations and Functions | KEY: Conceptual Understanding |

3. **ANS: C**  
   Recall radical definition: $\sqrt[m]{x^n} = x^{\frac{m}{n}}$  
   
   | PTS: 1 | DIF: Easy | REF: 5.4 Logarithms and the Logarithmic Function | LOC: 12.RF7 | TOP: Relations and Functions | KEY: Conceptual Understanding | Procedural Knowledge |

4. **ANS: D**  
   If there are no other transformations, we are looking for $base > 1$  
   
   | PTS: 1 | DIF: Easy | REF: 5.2 Analyzing Exponential Functions | LOC: 12.RF9 | TOP: Relations and Functions | KEY: Conceptual Understanding |

5. **ANS: B**  
   If there are no other transformations, we are looking for $base < 1$  
   
   | PTS: 1 | DIF: Easy | REF: 5.2 Analyzing Exponential Functions | LOC: 12.RF9 | TOP: Relations and Functions | KEY: Conceptual Understanding |

6. **ANS: C**  
   Look at all statements to ensure correctness.  
   
   | PTS: 1 | DIF: Easy | REF: 5.2 Analyzing Exponential Functions | LOC: 12.RF9 | TOP: Relations and Functions | KEY: Conceptual Understanding |

7. **ANS: B**  
   Use logs to solve for exponents, then equate the exponents.  
   
   | PTS: 1 | DIF: Easy | REF: 5.3 Solving Exponential Equations | LOC: 12.RF10 | TOP: Relations and Functions | KEY: Procedural Knowledge |

8. **ANS: D**  
   Consider all statements so you don’t miss the correct answer.  
   
   | PTS: 1 | DIF: Easy | REF: 5.4 Logarithms and the Logarithmic Function | LOC: 12.RF9 | TOP: Relations and Functions | KEY: Conceptual Understanding |
10. ANS: B PTS: 1 DIF: Easy REF: 5.5 The Laws of Logarithms
    LOC: 12.RF8 TOP: Relations and Functions KEY: Procedural Knowledge
11. ANS: C PTS: 1 DIF: Easy REF: 5.5 The Laws of Logarithms
    LOC: 12.RF8 TOP: Relations and Functions KEY: Procedural Knowledge
12. ANS: A PTS: 1 DIF: Easy
    REF: 5.7 Solving Logarithmic and Exponential Equations LOC: 12.RF8
    TOP: Relations and Functions KEY: Procedural Knowledge | Conceptual Understanding
13. ANS: C PTS: 1 DIF: Easy
    REF: 5.7 Solving Logarithmic and Exponential Equations LOC: 12.RF10
    TOP: Relations and Functions KEY: Procedural Knowledge
14. ANS: D
    The untransformed range of exponentials is $y > 0$. Transforms that affect the range are vertical reflection and shift.
    PTS: 1 DIF: Moderate REF: 5.2 Analyzing Exponential Functions
    LOC: 12.RF9 TOP: Relations and Functions KEY: Conceptual Understanding
15. ANS: D
    The untransformed equation of the horizontal asymptote of an exponential is $y = 0$. The horizontal asymptote is affected by the vertical shift.
    PTS: 1 DIF: Moderate REF: 5.2 Analyzing Exponential Functions
    LOC: 12.RF9 TOP: Relations and Functions KEY: Conceptual Understanding
16. ANS: C
    Use logs to solve for exponents.
    Recall radical definition: $\sqrt[\frac{m}{n}]{x^n} = x^{\frac{m}{n}}$
    PTS: 1 DIF: Moderate REF: 5.3 Solving Exponential Equations
    LOC: 12.RF9 TOP: Relations and Functions KEY: Procedural Knowledge
17. ANS: D
    Make sure you identify the correct variables to get the proper transform.
    PTS: 1 DIF: Moderate REF: 5.2 Analyzing Exponential Functions
    LOC: 12.RF9 TOP: Relations and Functions KEY: Procedural Knowledge
18. ANS: A
    Consider all the statements to ensure correctness.
    PTS: 1 DIF: Moderate REF: 5.6 Analyzing Logarithmic Functions
    LOC: 12.RF9 TOP: Relations and Functions KEY: Conceptual Understanding
19. ANS: C
    The x-intercepts are affected by all transforms, so solve for $0 = 3 \log_2(x - 4)$
    PTS: 1 DIF: Moderate REF: 5.6 Analyzing Logarithmic Functions
    LOC: 12.RF9 TOP: Relations and Functions KEY: Procedural Knowledge | Conceptual Understanding
20. ANS: A  
Recall the domain is affected by horizontal scale and shift.

PTS: 1  DIF: Moderate  REF: 5.6 Analyzing Logarithmic Functions  
LOC: 12.RF9  TOP: Relations and Functions  
KEY: Procedural Knowledge | Conceptual Understanding

21. ANS: A  
\[ x \approx \frac{\log 324}{\log 2 \cdot 3^4} \]  
Algebraically:  

PTS: 1  DIF: Moderate  REF: 5.7 Solving Logarithmic and Exponential Equations  
LOC: 12.RF10  TOP: Relations and Functions  
KEY: Procedural Knowledge | Conceptual Understanding

22. ANS: D  
Recall  

\[ I = 10^{\frac{L}{10}} I_0 \]  

PTS: 1  DIF: Moderate  REF: 5.8 Solving Problems with Exponents and Logarithms  
LOC: 12.RF10  TOP: Relations and Functions  
KEY: Conceptual Understanding | Procedural Knowledge

23. ANS: A  
PTS: 1  DIF: Moderate  REF: 5.8 Solving Problems with Exponents and Logarithms  
LOC: 12.RF10  TOP: Relations and Functions  
KEY: Conceptual Understanding | Procedural Knowledge

24. ANS: B  
It is best to do all the algebra before using calculator to avoid rounding errors.

PTS: 1  DIF: Moderate  REF: 5.8 Solving Problems with Exponents and Logarithms  
LOC: 12.RF10  TOP: Relations and Functions  
KEY: Conceptual Understanding | Procedural Knowledge

SHORT ANSWER

25. ANS:  
-0.5 if writing decimals instead of logarithms.  
\log_5 15626, \log_4 14234, \log_2 171, \log_3 6280

PTS: 1  DIF: Easy  REF: 5.6 Analyzing Logarithmic Functions  
LOC: 12.RF9  TOP: Relations and Functions  
KEY: Procedural Knowledge

26. ANS:  
-1 if not using an equation.  
Find corresponding points and solve equation.  
Easy grid points are \((3, 1)\) and \((3, -4)\).  
\[ y = -4 \log_3 x \]

PTS: 1  DIF: Easy  REF: 5.6 Analyzing Logarithmic Functions  
LOC: 12.RF9  TOP: Relations and Functions  
KEY: Conceptual Understanding
27. ANS: 
\[ x = 6 \] is a root of the equation.

PTS: 1  DIF: Easy  REF: 5.7 Solving Logarithmic and Exponential Equations
LOC: 12.RF10  TOP: Relations and Functions
KEY: Conceptual Understanding | Procedural Knowledge

28. ANS: 
Use 2 equations and find the intersect. Or use 1 equation and find the root or zero. 
\[ x = 2.8 \]

PTS: 1  DIF: Easy  REF: 5.3 Solving Exponential Equations
LOC: 12.RF10  TOP: Relations and Functions
KEY: Procedural Knowledge

29. ANS: 
\[ x = -1 \]

PTS: 1  DIF: Easy  REF: 5.6 Analyzing Logarithmic Functions
LOC: 12.RF9  TOP: Relations and Functions
KEY: Conceptual Understanding | Procedural Knowledge

30. ANS: 
Determine a common base. Use logs to solve for exponents, then equate the exponents. 1 mark will be lost for just using the calculator. 
\[ x = 4 \text{ or } x = -1 \]

PTS: 1  DIF: Moderate  REF: 5.3 Solving Exponential Equations
LOC: 12.RF10  TOP: Relations and Functions
KEY: Procedural Knowledge

31. ANS: 
Determine a common base. Use logs to solve for exponents, then equate the exponents. 1 mark will be lost for just using the calculator. 
\[ x = \frac{24}{7} \]

PTS: 1  DIF: Moderate  REF: 5.3 Solving Exponential Equations
LOC: 12.RF10  TOP: Relations and Functions
KEY: Procedural Knowledge

32. ANS: 
Recall: Check for extraneous roots. 
\[ x = 14 \]

PTS: 1  DIF: Moderate  REF: 5.7 Solving Logarithmic and Exponential Equations
LOC: 12.RF10  TOP: Relations and Functions
KEY: Procedural Knowledge | Conceptual Understanding

33. ANS: 
It is best to do all the algebra before using calculator to avoid rounding errors. 
-.5 for rounding errors and missing units, rounding up is also acceptable. 
Raj will have $3869.07.

PTS: 1  DIF: Moderate  REF: 5.8 Solving Problems with Exponents and Logarithms
LOC: 12.RF10  TOP: Relations and Functions
KEY: Procedural Knowledge | Conceptual Understanding
34. ANS: 
-1 if not doing linear interpolation.
\[ 2^3 = 8, \ 2^4 = 16 \]
\[ 2^3 < 14.3 < 2^4 \]
\[ \log_2 14.3 \approx 3 \frac{14.3 - 8}{16 - 8} = \frac{63}{80} \]

PTS: 1  DIF: Difficult  REF: 5.4 Logarithms and the Logarithmic Function
LOC: 12.RF7  TOP: Relations and Functions
KEY: Conceptual Understanding | Procedural Knowledge

PROBLEM

35. ANS: 
-.5 if rounded incorrectly and/or missing units
a) Each reflection reduces the intensity of the light by 10%. So, 90% of the light’s intensity remains.
   For 0 mirrors, the percent of light is: \( P = 100 \)
   For 1 mirror, the percent of light is: \( A = 100(0.9), \) or 90
   For 2 mirrors, the percent of light is: \( A = 100(0.9)^2, \) or 81
   For 3 mirrors, the percent of light is: \( A = 100(0.9)^3, \) or 72.9
   For \( n \) mirrors, the percent of light is: \( A = 100(0.9)^n \)

b) Solve the equation: \( 53 = 100(0.9)^n \)
   Graph a related function: \( y = 100(0.9)^x - 53 \)
   The approximate zero of the function is: \( 6.02577 \)
   So, the intensity of a beam of light is reduced to approximately 53% when it has been reflected in 6 mirrors.

PTS: 1  DIF: Moderate  REF: 5.3 Solving Exponential Equations
LOC: 12.RF10  TOP: Relations and Functions
KEY: Communication | Conceptual Understanding | Problem-Solving Skills
36. **ANS:**
It is best to do all the algebra before using calculator to avoid rounding errors.
-.5 for rounding errors
Recall: $[H^+] = 10^{-pH}$
a) Use the equation: $pH = -\log[H^+]$ Substitute $[H^+] = 7.9 \times 10^{-6}$.
$$pH = -\log(7.9 \times 10^{-6})$$
$$pH \approx 5.1$$
The watermelon has a pH of approximately 5.1.
b) Use the equation: $pH = -\log[H^+]$ Substitute pH = 2.2.
$$2.2 = -\log[H^+]$$
$$-2.2 = \log[H^+]$$
$$10^{-2.2} = [H^+]$$
$$[H^+] \approx 0.0063$$
The hydrogen-ion concentration of the lemon juice is approximately 0.0063 moles/litre.

37. **ANS:**
-.5 if rounded incorrectly and/or missing units
a) The value of a car depreciates by 14% each year.
So, each year, the car is worth 86% of its value in the previous year.
After 0 years, the car’s value as a percent of its initial value is: $P = 100$
After 1 year, the car’s value as a percent of its initial value is: $P = 100(0.86)$, or 86
After 2 years, the car’s value as a percent of its initial value is: $P = 100(0.86)^2$, or 73.96
After 3 years, the car’s value as a percent of its initial value is: $P = 100(0.86)^3$, or 63.6056
After $t$ years, the car’s value as a percent of its initial value is: $P = 100(0.86)^t$
b) Solve: $25 = 100(0.86)^t$
Graph a related function: $y = 100(0.86)^t - 25$
The approximate zero of the function is: 9.19154
So, the car is worth 25% of its initial value after about 9 years.
ANS:
Write $\frac{270}{49}$ in terms of a power of 3, a power of 7, and a power of 10.

\[
\log\left(\frac{270}{49}\right) = \log\left(\frac{3^3 \cdot 10}{7^2}\right)
\]

\[
= \log(3^3) + \log(10) - \log(7^2)
\]

\[
= 3 \log(3) + \log(10) - 2 \log(7)
\]

\[
\approx 3(0.4771) + 1 - 2(0.8451)
\]

\[
\approx 0.7411
\]
ANS: It is best to do all the algebra before using calculator to avoid rounding errors. -.5 for rounding errors and missing units

Recall: \( I = 10^M S \)

Use the equation \( M = \log\left( \frac{I}{S} \right) \) to determine the intensity of an earthquake with magnitude 5.1.

Substitute \( M = 5.1 \).

\[
5.1 = \log\left( \frac{I}{S} \right)
\]

\[
10^{5.1} = \frac{I}{S}
\]

\[
I = 10^{5.1} S
\]

An earthquake that is one-third as intense has intensity: \( I = \frac{1}{3} \left( 10^{5.1} S \right) \)

Use the equation: \( M = \log\left( \frac{I}{S} \right) \) Substitute \( I = \frac{1}{3} \left( 10^{5.1} S \right) \).

\[
M = \log\left( \frac{\frac{1}{3} \left( 10^{5.1} S \right)}{S} \right)
\]

\[
M = \log\left( \frac{1}{3} \left( 10^{5.1} \right) \right)
\]

\[
M \approx 4.6
\]

The magnitude is approximately 4.6.
40. ANS:
Write both powers with the same base, then equate the exponents, and simplify the equation.
-1.5 if just using calculator.
\[4^x = 32^{(x^2 + k)}\]
\[2^{2x} = 2^{5(x^2 + k)}\]
\[2x = 5x^2 + 5k\]
\[0 = 5x^2 - 2x + 5k\]
This is a quadratic equation.
It has no real roots when the discriminant is less than 0.
Determine when the discriminant is less than 0.
\[(-2)^2 - 4(5)(5k) < 0\]
\[4 < 100k\]
\[\frac{1}{25} < k\]
So, the equation \(4^x = 32^{(x^2 + k)}\) has no real solution when \(k > \frac{1}{25}\).

PTS: 1  DIF: Difficult  REF: 5.3 Solving Exponential Equations
LOC: 12.RF10  TOP: Relations and Functions
KEY: Conceptual Understanding | Problem-Solving Skills

41. ANS:
a) Count: 21
\[\frac{n-21}{12}\]
b) Substitute: \(Y(n) = 55 \cdot 2^\frac{n-21}{12}\)
c) Semitone \(G# = 9\)
Determine the note number: \((9 - 1) + 12 \cdot 2 = 32\)
Substitute: \(Y(32) = 55 \cdot 2^{\frac{32-21}{12}} = 103.83\) Hz

PTS: 1  DIF: Difficult  REF: 5.8 Solving Log and Exp Problems