PreCalc 12 Chapter 5 Review 2017 v1
Answer Section

MULTIPLE CHOICE

1. ANS: D
Use substitution: 2^0

PTS: 1 DIF: Easy REF: 5.1 Math Lab: Graphing Exponential Functions
LOC: 12.RF9 TOP: Relations and Functions KEY: Procedural Knowledge

2. ANS: A
The untransformed range of exponentials is $y > 0$

PTS: 1 DIF: Easy REF: 5.1 Math Lab: Graphing Exponential Functions
LOC: 12.RF9 TOP: Relations and Functions KEY: Conceptual Understanding

3. ANS: C
Recall radical definition: $\sqrt[n]{x^m} = x^{\frac{m}{n}}$

PTS: 1 DIF: Easy REF: 5.4 Logarithms and the Logarithmic Function
LOC: 12.RF7 TOP: Relations and Functions
KEY: Conceptual Understanding | Procedural Knowledge

4. ANS: D
If there are no other transformations, we are looking for $base > 1$

PTS: 1 DIF: Easy REF: 5.2 Analyzing Exponential Functions
LOC: 12.RF9 TOP: Relations and Functions KEY: Conceptual Understanding

5. ANS: B
If there are no other transformations, we are looking for $base < 1$

PTS: 1 DIF: Easy REF: 5.2 Analyzing Exponential Functions
LOC: 12.RF9 TOP: Relations and Functions KEY: Conceptual Understanding

6. ANS: C
Look at all statements to ensure correctness.

PTS: 1 DIF: Easy REF: 5.2 Analyzing Exponential Functions
LOC: 12.RF9 TOP: Relations and Functions KEY: Conceptual Understanding

7. ANS: B
Use logs to solve for exponents, then equate the exponents.

PTS: 1 DIF: Easy REF: 5.3 Solving Exponential Equations
LOC: 12.RF10 TOP: Relations and Functions KEY: Procedural Knowledge

8. ANS: D
Consider all statements so you don't miss the correct answer.

PTS: 1 DIF: Easy REF: 5.4 Logarithms and the Logarithmic Function
LOC: 12.RF9 TOP: Relations and Functions KEY: Conceptual Understanding

9. ANS: A PTS: 1 DIF: Easy
REF: 5.4 Logarithms and the Logarithmic Function LOC: 12.RF7
TOP: Relations and Functions KEY: Conceptual Understanding | Procedural Knowledge

10. ANS: B PTS: 1 DIF: Easy REF: 5.5 The Laws of Logarithms
LOC: 12.RF8 TOP: Relations and Functions KEY: Procedural Knowledge

11. ANS: C PTS: 1 DIF: Easy REF: 5.5 The Laws of Logarithms
LOC: 12.RF8 TOP: Relations and Functions KEY: Procedural Knowledge

12. ANS: A PTS: 1 DIF: Easy
REF: 5.7 Solving Logarithmic and Exponential Equations LOC: 12.RF8
TOP: Relations and Functions KEY: Procedural Knowledge | Conceptual Understanding

13. ANS: C PTS: 1 DIF: Easy
REF: 5.7 Solving Logarithmic and Exponential Equations LOC: 12.RF10
TOP: Relations and Functions KEY: Procedural Knowledge

14. ANS: D
The untransformed range of exponentials is $y > 0$. Transforms that affect the range are vertical reflection and shift.

PTS: 1 DIF: Moderate REF: 5.2 Analyzing Exponential Functions
LOC: 12.RF9 TOP: Relations and Functions KEY: Conceptual Understanding

15. ANS: D
The untransformed equation of the horizontal asymptote of an exponential is $y = 0$. The horizontal asymptote is affected by the vertical shift.

PTS: 1 DIF: Moderate REF: 5.2 Analyzing Exponential Functions
LOC: 12.RF9 TOP: Relations and Functions KEY: Conceptual Understanding

16. ANS: C
Use logs to solve for exponents.

Recall radical definition: $\sqrt[n]{x^m} = x^{\frac{m}{n}}$

PTS: 1 DIF: Moderate REF: 5.3 Solving Exponential Equations
LOC: 12.RF9 TOP: Relations and Functions KEY: Procedural Knowledge

17. ANS: D
Make sure you identify the correct variables to get the proper transform.

PTS: 1 DIF: Moderate REF: 5.2 Analyzing Exponential Functions
LOC: 12.RF9 TOP: Relations and Functions KEY: Conceptual Understanding

18. ANS: A
Consider all the statements to ensure correctness.

PTS: 1 DIF: Moderate REF: 5.6 Analyzing Logarithmic Functions
LOC: 12.RF9 TOP: Relations and Functions KEY: Conceptual Understanding

19. ANS: C
The x-intercepts are affected by all transforms, so solve for $0 = 3\log_2(x-4)$

PTS: 1 DIF: Moderate REF: 5.6 Analyzing Logarithmic Functions
LOC: 12.RF9 TOP: Relations and Functions
KEY: Procedural Knowledge | Conceptual Understanding

20. ANS: A
Recall the domain is affected by horizontal scale and shift.

PTS: 1 DIF: Moderate REF: 5.6 Analyzing Logarithmic Functions
LOC: 12.RF9 TOP: Relations and Functions
KEY: Procedural Knowledge | Conceptual Understanding

21. ANS: A
Algebraically: $x \approx \dfrac{\log \dfrac{324}{2 \cdot 3^4}}{\log 3}$

PTS: 1 DIF: Moderate REF: 5.7 Solving Logarithmic and Exponential Equations
LOC: 12.RF10 TOP: Relations and Functions
KEY: Procedural Knowledge | Conceptual Understanding

22. ANS: D
Recall $I = 10^{\frac{L}{10}} I_0$

PTS: 1 DIF: Moderate REF: 5.8 Solving Problems with Exponents and Logarithms
LOC: 12.RF10 TOP: Relations and Functions
KEY: Conceptual Understanding | Procedural Knowledge

23. ANS: A PTS: 1 DIF: Moderate
REF: 5.8 Solving Problems with Exponents and Logarithms LOC: 12.RF10
TOP: Relations and Functions KEY: Conceptual Understanding | Procedural Knowledge

24. ANS: B
It is best to do all the algebra before using calculator to avoid rounding errors.

PTS: 1 DIF: Moderate REF: 5.8 Solving Problems with Exponents and Logarithms
LOC: 12.RF10 TOP: Relations and Functions
KEY: Conceptual Understanding | Procedural Knowledge

SHORT ANSWER

25. ANS:
-.5 if writing decimals instead of logarithms.
$\log_5 15626$, $\log_4 14234$, $\log_2 171$, $\log_3 6280$

PTS: 1 DIF: Easy REF: 5.6 Analyzing Logarithmic Functions
LOC: 12.RF9 TOP: Relations and Functions KEY: Procedural Knowledge

26. ANS:
-1 for not using an equation.
Find corresponding points and solve equation.
Easy grid points are $(3,1)$ and $(3,-4)$.
$y = -4\log_3 x$

PTS: 1 DIF: Easy REF: 5.6 Analyzing Logarithmic Functions
LOC: 12.RF9 TOP: Relations and Functions KEY: Conceptual Understanding

27. ANS:
$x = 6$ is a root of the equation.

PTS: 1 DIF: Easy REF: 5.7 Solving Logarithmic and Exponential Equations
LOC: 12.RF10 TOP: Relations and Functions
KEY: Conceptual Understanding | Procedural Knowledge

28. ANS:
Use 2 equations and find the intersect. Or use 1 equation and find the root or zero.
$x \doteq 2.8$

PTS: 1 DIF: Easy REF: 5.3 Solving Exponential Equations
LOC: 12.RF10 TOP: Relations and Functions KEY: Procedural Knowledge

29. ANS:
$x = -1$

PTS: 1 DIF: Easy REF: 5.6 Analyzing Logarithmic Functions
LOC: 12.RF9 TOP: Relations and Functions
KEY: Conceptual Understanding | Procedural Knowledge

30. ANS:
Determine a common base. Use logs to solve for exponents, then equate the exponents. 1 mark will be lost for just using the calculator.
$x = 4$ or $x = -1$

PTS: 1 DIF: Moderate REF: 5.3 Solving Exponential Equations
LOC: 12.RF10 TOP: Relations and Functions KEY: Procedural Knowledge

31. ANS:
Determine a common base. Use logs to solve for exponents, then equate the exponents. 1 mark will be lost for just using the calculator.
$x = \frac{24}{7}$

PTS: 1 DIF: Moderate REF: 5.3 Solving Exponential Equations
LOC: 12.RF10 TOP: Relations and Functions KEY: Procedural Knowledge

32. ANS:
Recall: Check for extraneous roots.
$x = 14$

PTS: 1 DIF: Moderate REF: 5.7 Solving Logarithmic and Exponential Equations
LOC: 12.RF10 TOP: Relations and Functions
KEY: Procedural Knowledge | Conceptual Understanding

33. ANS:
It is best to do all the algebra before using calculator to avoid rounding errors.
-.5 for rounding errors and missing units, rounding up is also acceptable.
Raj will have $3869.07.

PTS: 1 DIF: Moderate REF: 5.8 Solving Problems with Exponents and Logarithms
LOC: 12.RF10 TOP: Relations and Functions
KEY: Procedural Knowledge | Conceptual Understanding

34. ANS:
-1 if not doing linear interpolation.
$2^3 = 8, 2^4 = 16$
$2^3 < 14.3 < 2^4$
$\log_2 14.3 \doteq 3\dfrac{14.3-8}{16-8} = 3\dfrac{63}{80}$

PTS: 1 DIF: Difficult REF: 5.4 Logarithms and the Logarithmic Function
LOC: 12.RF7 TOP: Relations and Functions
KEY: Conceptual Understanding | Procedural Knowledge

PROBLEM

35. ANS:
-.5 if rounded incorrectly and/or missing units

a) Each reflection reduces the intensity of the light by 10%. So, 90% of the light's intensity remains.
For 0 mirrors, the percent of light is: $P = 100$
For 1 mirror, the percent of light is: $A = 100(0.9)$, or 90
For 2 mirrors, the percent of light is: $A = 100(0.9)^2$, or 81
For 3 mirrors, the percent of light is: $A = 100(0.9)^3$, or 72.9
For n mirrors, the percent of light is: $A = 100(0.9)^n$

b) Solve the equation: $53 = 100(0.9)^n$
Graph a related function: $y = 100(0.9)^x - 53$
The approximate zero of the function is: 6.02577
So, the intensity of a beam of light is reduced to approximately 53% when it has been reflected in 6 mirrors.

PTS: 1 DIF: Moderate REF: 5.3 Solving Exponential Equations
LOC: 12.RF10 TOP: Relations and Functions
KEY: Communication | Conceptual Understanding | Problem-Solving Skills

36. ANS:
It is best to do all the algebra before using calculator to avoid rounding errors.
-.5 for rounding errors
Recall: $\left[H^+\right] = 10^{-pH}$

a) Use the equation: $\text{pH} = -\log\left[\text{H}^+\right]$ Substitute $\left[\text{H}^+\right] = 7.9 \times 10^{-6}$.

$\text{pH} = -\log(7.9 \times 10^{-6})$

$\text{pH} \doteq 5.1$

The watermelon has a pH of approximately 5.1.

b) Use the equation: $\text{pH} = -\log\left[\text{H}^+\right]$ Substitute $\text{pH} = 2.2$.

$2.2 = -\log\left[\text{H}^+\right]$

$-2.2 = \log\left[\text{H}^+\right]$

$10^{-2.2} = \left[\text{H}^+\right]$

$\left[\text{H}^+\right] \doteq 0.0063$

The hydrogen-ion concentration of the lemon juice is approximately 0.0063 moles/litre.

PTS: 1 DIF: Moderate REF: 5.8 Solving Problems with Exponents and Logarithms
LOC: 12.RF10 TOP: Relations and Functions KEY: Procedural Knowledge

37. ANS:
-.5 if rounded incorrectly and/or missing units

a) The value of a car depreciates by 14% each year.
So, each year, the car is worth 86% of its value in the previous year.
After 0 years, the car's value as a percent of its initial value is: $P = 100$
After 1 year, the car's value as a percent of its initial value is: $P = 100(0.86)$, or 86
After 2 years, the car's value as a percent of its initial value is: $P = 100(0.86)^2$, or 73.96
After 3 years, the car's value as a percent of its initial value is: $P = 100(0.86)^3$, or 63.6056
After t years, the car's value as a percent of its initial value is: $P = 100(0.86)^t$

b) Solve: $25 = 100(0.86)^t$
Graph a related function: $y = 100(0.86)^t - 25$
The approximate zero of the function is: 9.19154
So, the car is worth 25% of its initial value after about 9 years.

PTS: 1 DIF: Moderate REF: 5.3 Solving Exponential Equations
LOC: 12.RF10 TOP: Relations and Functions
KEY: Conceptual Understanding | Problem-Solving Skills | Communication

38. ANS:

Write $\frac{270}{49}$ in terms of a power of 3, a power of 7, and a power of 10.

$$\log\left(\frac{270}{49}\right) = \log\left(\frac{3^3 \cdot 10}{7^2}\right)$$

$$= \log(3^3) + \log(10) - \log(7^2)$$

$$= 3\log(3) + \log(10) - 2\log(7)$$

$$\doteq 3(0.4771) + 1 - 2(0.8451)$$

$$\doteq 0.7411$$

PTS: 1 DIF: Moderate REF: 5.5 The Laws of Logarithms
LOC: 12.RF8 TOP: Relations and Functions
KEY: Conceptual Understanding | Problem-Solving Skills

39. ANS:
It is best to do all the algebra before using calculator to avoid rounding errors.
-.5 for rounding errors and missing units
Recall: $I = 10^M S$

Use the equation $M = \log\left(\frac{I}{S}\right)$ to determine the intensityof an earthquake with magnitude 5.1.

Substitute $M = 5.1$.

$$5.1 = \log\left(\frac{I}{S}\right)$$

$$10^{5.1} = \frac{I}{S}$$

$$I = 10^{5.1} S$$

An earthquake that is one-third as intense has intensity: $I = \frac{1}{3}\left(10^{5.1} S\right)$

Use the equation: $M = \log\left(\frac{I}{S}\right)$ Substitute $I = \frac{1}{3}\left(10^{5.1} S\right)$.

$$M = \log\left(\frac{\frac{1}{3}\left(10^{5.1} S\right)}{S}\right)$$

$$M = \log\left(\frac{1}{3}\left(10^{5.1}\right)\right)$$

$M \doteq 4.6$
The magnitude is approximately 4.6.

PTS: 1 DIF: Moderate REF: 5.8 Solving Problems with Exponents and Logarithms
LOC: 12.RF10 TOP: Relations and Functions
KEY: Problem-Solving Skills | Conceptual Understanding | Procedural Knowledge

40. ANS:
Write both powers with the same base, then equate the exponents, and simplify the equation.
-1.5 if just using calculator.

$4^x = 32^{(x^2 + k)}$

$2^{2x} = 2^{5(x^2 + k)}$

$2x = 5x^2 + 5k$

$0 = 5x^2 - 2x + 5k$

This is a quadratic equation.
It has no real roots when the discriminant is less than 0.
Determine when the discriminant is less than 0.

$(-2)^2 - 4(5)(5k) < 0$

$(-2)^2 < 4(5)(5k)$

$4 < 100k$

$\frac{1}{25} < k$

So, the equation $4^x = 32^{(x^2 + k)}$ has no real solution when $k > \frac{1}{25}$.

PTS: 1 DIF: Difficult REF: 5.3 Solving Exponential Equations
LOC: 12.RF10 TOP: Relations and Functions
KEY: Conceptual Understanding | Problem-Solving Skills

41. ANS:

a) Count: 21

b) Substitute: $Y(n) = 55 \cdot 2^{\frac{n-21}{12}}$

c) Semitone $G\# = 9$
Determine the note number: $(9 - 1) + 12 \cdot 2 = 32$
Substitute: $Y(32) = 55 \cdot 2^{\frac{32-21}{12}} = 103.83$ Hz

PTS: 1 DIF: Difficult REF: 5.8 Solving Log and Exp Problems