

PreCalc 12 Chapter 7 Review 2017 v1 Answer Section

MULTIPLE CHOICE

1. ANS: B

Use mnemonic ASTC

PTS: 1 DIF: Easy REF: 7.1 Solving Trigonometric Equations Graphically
LOC: 12.T5 TOP: Trigonometry KEY: Procedural Knowledge

2. ANS: B

Recall that you must adjust the period.

PTS: 1 DIF: Easy REF: 7.1 Solving Trigonometric Equations Graphically
LOC: 12.T5 TOP: Trigonometry
KEY: Conceptual Understanding | Procedural Knowledge

3. ANS: C

The non-solution will have a different denominator.

PTS: 1 DIF: Easy REF: 7.2 Solving Trigonometric Equations Algebraically
LOC: 12.T5 TOP: Trigonometry KEY: Procedural Knowledge

4. ANS: B

The non-solution will have a different denominator.

PTS: 1 DIF: Easy REF: 7.2 Solving Trigonometric Equations Algebraically
LOC: 12.T5 TOP: Trigonometry KEY: Procedural Knowledge

5. ANS: C

Recall your special angles and mnemonic ASTC.

PTS: 1 DIF: Easy REF: 7.2 Solving Trigonometric Equations Algebraically
LOC: 12.T5 TOP: Trigonometry
KEY: Conceptual Understanding | Procedural Knowledge

6. ANS: B

Recall the basic identities and cancel.

PTS: 1 DIF: Easy REF: 7.3 Reciprocal and Quotient Identities
LOC: 12.T6 TOP: Trigonometry KEY: Procedural Knowledge

7. ANS: D

Recall the basic identities and cancel.

PTS: 1 DIF: Easy REF: 7.3 Reciprocal and Quotient Identities
LOC: 12.T6 TOP: Trigonometry KEY: Procedural Knowledge

8. ANS: B

Recall the basic identities and cancel.

PTS: 1 DIF: Easy REF: 7.3 Reciprocal and Quotient Identities
LOC: 12.T6 TOP: Trigonometry KEY: Procedural Knowledge

9. ANS: C

Recall the basic identities and check the denominator for NPV's

PTS: 1 DIF: Easy REF: 7.3 Reciprocal and Quotient Identities
 LOC: 12.T6 TOP: Trigonometry KEY: Conceptual Understanding

10. ANS: A

Recall the basic and Pythagorean identities and check the denominator for NPV's.

PTS: 1 DIF: Easy REF: 7.4 The Pythagorean Identities
 LOC: 12.T6 TOP: Trigonometry KEY: Procedural Knowledge

11. ANS: C

Recall the sum/difference identities then simplify.

PTS: 1 DIF: Easy REF: 7.5 Sum and Difference Identities
 LOC: 12.T6 TOP: Trigonometry KEY: Procedural Knowledge

12. ANS: A

Recall the double-angle identities, special angles, ASTC, and solve.

PTS: 1 DIF: Easy REF: 7.6 Double-Angle Identities
 LOC: 12.T5 TOP: Trigonometry
 KEY: Conceptual Understanding | Procedural Knowledge

13. ANS: C

Recall the double-angle identities then simplify.

PTS: 1 DIF: Easy REF: 7.6 Double-Angle Identities
 LOC: 12.T6 TOP: Trigonometry KEY: Procedural Knowledge

14. ANS: A

Recall the double-angle identities, special angles, ASTC, and solve.

PTS: 1 DIF: Easy REF: 7.6 Double-Angle Identities
 LOC: 12.T5 TOP: Trigonometry
 KEY: Conceptual Understanding | Procedural Knowledge

15. ANS: B

Recall the double-angle identities, special angles, ASTC, and solve.

PTS: 1 DIF: Easy REF: 7.6 Double-Angle Identities
 LOC: 12.T5 TOP: Trigonometry
 KEY: Conceptual Understanding | Procedural Knowledge

16. ANS: A

Use graphing calculator to solve for roots.

Not as good to use TRACE because of higher order trig.

PTS: 1 DIF: Moderate REF: 7.1 Solving Trigonometric Equations Graphically
 LOC: 12.T5 TOP: Trigonometry KEY: Procedural Knowledge

17. ANS: D

Eliminate answer that are not in the domain.
Use graphing calculator to solve for roots.
Not as good to use TRACE because of higher order trig.

PTS: 1 DIF: Moderate REF: 7.1 Solving Trigonometric Equations Graphically
LOC: 12.T5 TOP: Trigonometry
KEY: Conceptual Understanding | Procedural Knowledge

18. ANS: C

If non-MC, you cannot use TRACE.
Quickly eliminate distractors that are out of the domain.
Use TRACE to eliminate answers that are not within +/- .1

PTS: 1 DIF: Moderate REF: 7.1 Solving Trigonometric Equations Graphically
LOC: 12.T5 TOP: Trigonometry
KEY: Conceptual Understanding | Procedural Knowledge

19. ANS: C

If non-MC, you cannot use TRACE.
Quickly eliminate distractors that are out of the domain.
Use TRACE to eliminate answers that are not within +/- .1

PTS: 1 DIF: Moderate REF: 7.1 Solving Trigonometric Equations Graphically
LOC: 12.T5 TOP: Trigonometry
KEY: Conceptual Understanding | Procedural Knowledge

20. ANS: A

Recall your special angles and mnemonic ASTC.

PTS: 1 DIF: Moderate REF: 7.2 Solving Trigonometric Equations Algebraically
LOC: 12.T5 TOP: Trigonometry
KEY: Conceptual Understanding | Procedural Knowledge

21. ANS: A

Recall the Pythagorean identities and simplify.

PTS: 1 DIF: Moderate REF: 7.4 The Pythagorean Identities
LOC: 12.T6 TOP: Trigonometry KEY: Procedural Knowledge

22. ANS: D

Recall the sum/difference identities, special angles, and simplify.

PTS: 1 DIF: Moderate REF: 7.5 Sum and Difference Identities
LOC: 12.T5 TOP: Trigonometry
KEY: Conceptual Understanding | Procedural Knowledge

23. ANS: A

Recall the sum/difference identities, special angles, and simplify.

PTS: 1 DIF: Moderate REF: 7.5 Sum and Difference Identities
LOC: 12.T5 TOP: Trigonometry
KEY: Conceptual Understanding | Procedural Knowledge

24. ANS: D

Recall the sum/difference identities, special angles, and simplify.

PTS: 1 DIF: Moderate REF: 7.5 Sum and Difference Identities

LOC: 12.T5 TOP: Trigonometry

KEY: Conceptual Understanding | Procedural Knowledge

25. ANS: A

Recall the sum/difference identities, special angles, number of solutions, ASTC, and simplify.

PTS: 1 DIF: Moderate REF: 7.5 Sum and Difference Identities

LOC: 12.T5 TOP: Trigonometry

KEY: Conceptual Understanding | Procedural Knowledge

26. ANS: A

Recall the sum/difference identities, special angles, number of solutions, ASTC, and simplify.

PTS: 1 DIF: Moderate REF: 7.5 Sum and Difference Identities

LOC: 12.T5 TOP: Trigonometry

KEY: Conceptual Understanding | Procedural Knowledge

SHORT ANSWER

27. ANS:

Recall that you must adjust the period.

-.5 for wrong period.

$$x \doteq 0.12 + \frac{2\pi}{5} k, k \in \mathbf{Z} \text{ or } x \doteq 1.14 + \frac{2\pi}{5} k, k \in \mathbf{Z}$$

PTS: 1 DIF: Easy REF: 7.1 Solving Trigonometric Equations Graphically

LOC: 12.T5 TOP: Trigonometry

KEY: Conceptual Understanding | Procedural Knowledge

28. ANS:

General solution must be in principal angle.

Recall the number of solutions, special angles and mnemonic ASTC.

-.5 for wrong period.

-.5 for not principal angle.

-.5 for not exact.

$$x = \frac{\pi}{2} + 2\pi k, k \in \mathbf{Z}$$

PTS: 1 DIF: Easy REF: 7.2 Solving Trigonometric Equations Algebraically

LOC: 12.T5 TOP: Trigonometry

KEY: Conceptual Understanding | Procedural Knowledge

29. ANS:

This is a trigonometric-polynomial equation. There are no general solutions.

Use calculator to solve for roots.

-.5 for missing solution.

-.5 for wrong rounding.

$$x \doteq -2.28 \text{ or } x \doteq 0.83$$

PTS: 1 DIF: Moderate REF: 7.1 Solving Trigonometric Equations Graphically

LOC: 12.T5 TOP: Trigonometry

KEY: Conceptual Understanding | Procedural Knowledge

30. ANS:

Use calculator to solve for roots.

-.5 for missing solution.

-.5 for not exact.

$$x = \frac{7}{3}\pi \text{ or } x = \frac{8}{3}\pi$$

PTS: 1 DIF: Moderate REF: 7.2 Solving Trigonometric Equations Algebraically

LOC: 12.T5 TOP: Trigonometry

KEY: Conceptual Understanding | Procedural Knowledge

31. ANS:

Recall the basic identities and number of solutions.

Use calculator to solve for roots AFTER using algebra to get simplified equation.

-.5 for missing solution.

-.5 for wrong rounding.

$$x \doteq 2.30 \text{ or } x \doteq 3.98$$

PTS: 1 DIF: Moderate REF: 7.3 Reciprocal and Quotient Identities

LOC: 12.T5 TOP: Trigonometry

KEY: Conceptual Understanding | Procedural Knowledge

32. ANS:

Recall the basic and Pythagorean identities and cancel.

-.5 for incorrect steps.

$$2\csc^5 \theta$$

PTS: 1 DIF: Moderate REF: 7.4 The Pythagorean Identities

LOC: 12.T6 TOP: Trigonometry

KEY: Conceptual Understanding | Procedural Knowledge

33. ANS:

Recall the sum/difference identities, special angles, ASTC.

-.5 for incorrect steps.

-.5 for not exact.

$$\frac{\sqrt{3} + 1}{2\sqrt{2}}$$

PTS: 1 DIF: Moderate REF: 7.5 Sum and Difference Identities

LOC: 12.T5 TOP: Trigonometry

KEY: Conceptual Understanding | Procedural Knowledge

34. ANS:

Recall the basic, sum/difference and Pythagorean identities.

-.5 for incorrect steps.

-.5 for not exact.

$$\cos(\alpha + \beta) = -\frac{319}{481}$$

PTS: 1 DIF: Moderate REF: 7.5 Sum and Difference Identities

LOC: 12.T5 TOP: Trigonometry

KEY: Procedural Knowledge | Conceptual Understanding

35. ANS:

Recall the double-angle and Pythagorean identities.

-.5 for incorrect steps.

-.5 for not exact.

$$\sin 2\theta = -\frac{240}{289}$$

PTS: 1 DIF: Moderate REF: 7.6 Double-Angle Identities

LOC: 12.T5 TOP: Trigonometry

KEY: Procedural Knowledge | Conceptual Understanding

36. ANS:

Recall the sum/difference identities.

-.5 for incorrect steps.

-.5 for not exact.

Amplitude: $\sqrt{2}$

PTS: 1 DIF: Difficult REF: 7.5 Sum and Difference Identities

LOC: 12.T4 TOP: Trigonometry

KEY: Problem-Solving Skills | Conceptual Understanding | Procedural Knowledge

PROBLEM

37. ANS:

-.5 for incorrect steps.

$$\text{R.S.} = \sec^2 \theta - \cos \theta \sec \theta$$

$$= \sec^2 \theta - 1$$

$$= \tan^2 \theta$$

$$= \text{L.S.}$$

The left side is equal to the right side, so the identity is proved.

PTS: 1 DIF: Easy REF: 7.4 The Pythagorean Identities

LOC: 12.T6 TOP: Trigonometry

KEY: Procedural Knowledge | Conceptual Understanding | Communication

38. ANS:

a) Rearrange the equation so that 0 is on one side: $2 + 8 \cos 5x = 0$ On a graphing calculator, determine the zeros of the function $y = 2 + 8 \cos 5x$ over the domain

$$0 \leq x < \frac{2\pi}{5}.$$

To the nearest hundredth, the solution in the domain $0 \leq x < \frac{2\pi}{5}$ is: $x \doteq 0.36$ or $x \doteq 0.89$

b) Identify the period of the function from the equation.

The period of $y = -2 - 8 \cos 5x$ is: $\frac{2\pi}{5}$ So, the general solution is approximately: $x \doteq 0.36 + \frac{2\pi}{5}k, k \in \mathbf{Z}$ or $x \doteq 0.89 + \frac{2\pi}{5}k, k \in \mathbf{Z}$

PTS: 1

DIF: Moderate

REF: 7.1 Solving Trigonometric Equations Graphically

LOC: 12.T5

TOP: Trigonometry

KEY: Conceptual Understanding | Procedural Knowledge | Communication

39. ANS:

$$-10 \sin^2 x + 11 \sin x = 3$$

$$10 \sin^2 x - 11 \sin x + 3 = 0$$

$$(5 \sin x - 3)(2 \sin x - 1) = 0$$

Either $5 \sin x - 3 = 0$ or $2 \sin x - 1 = 0$

$$5 \sin x - 3 = 0$$

$$\sin x = \frac{3}{5}$$

The reference angle is:

$$\sin^{-1}\left(\frac{3}{5}\right) = 36.8699\dots^\circ$$

The terminal arm of angle x in the domain $0 \leq x \leq 360^\circ$ lies in Quadrant 1 or Quadrant 2.In Quadrant 1, $x = 36.8699\dots^\circ$

$$\doteq 37^\circ$$

In Quadrant 2, $x = 180^\circ - 36.8699\dots^\circ$

$$\doteq 143^\circ$$

$$2 \sin x - 1 = 0$$

$$\sin x = \frac{1}{2}$$

The reference angle is:

$$\sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$$

The terminal arm of angle x in the domain $0 \leq x \leq 360^\circ$ lies in Quadrant 1 or Quadrant 2.In Quadrant 1, $x = 30^\circ$ In Quadrant 2, $x = 180^\circ - 30^\circ$

$$= 150^\circ$$

The roots are: $x = 30^\circ, x \doteq 37^\circ, x \doteq 143^\circ$, and $x = 150^\circ$

PTS: 1

DIF: Moderate

REF: 7.2 Solving Trigonometric Equations Algebraically

LOC: 12.T5

TOP: Trigonometry

KEY: Conceptual Understanding | Procedural Knowledge | Communication

40. ANS:

$$\text{L.S.} = \cot^2 \theta \sin \theta$$

$$= \left(\frac{\cos^2 \theta}{\sin^2 \theta} \right) \sin \theta$$

$$= \frac{\cos^2 \theta}{\sin \theta}$$

$$\text{R.S.} = \frac{2 \csc \theta}{\sec^2 \theta} - \cos \theta \cot \theta$$

$$= \frac{\left(\frac{2}{\sin \theta} \right)}{\left(\frac{1}{\cos^2 \theta} \right)} - \cos \theta \cdot \frac{\cos \theta}{\sin \theta}$$

$$= \frac{2}{\sin \theta} \cdot \cos^2 \theta - \frac{\cos^2 \theta}{\sin \theta}$$

$$= \frac{2 \cos^2 \theta - \cos^2 \theta}{\sin \theta}$$

$$= \frac{\cos^2 \theta}{\sin \theta}$$

The left and right sides simplify to the same expression, so the identity is proved.

PTS: 1 DIF: Moderate REF: 7.3 Reciprocal and Quotient Identities

LOC: 12.T6 TOP: Trigonometry

KEY: Procedural Knowledge | Conceptual Understanding | Communication

41. ANS:

$$\begin{aligned}
\text{L.S.} &= \tan\left(\frac{\pi}{4} + \theta\right) + \tan\left(\frac{\pi}{4} - \theta\right) \\
&= \frac{\tan\frac{\pi}{4} + \tan\theta}{1 - \tan\frac{\pi}{4}\tan\theta} + \frac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4}\tan\theta} \\
&= \frac{1 + \tan\theta}{1 - \tan\theta} + \frac{1 - \tan\theta}{1 + \tan\theta} \\
&= \frac{1 + \tan\theta}{1 - \tan\theta} \cdot \frac{1 + \tan\theta}{1 + \tan\theta} + \frac{1 - \tan\theta}{1 + \tan\theta} \cdot \frac{1 - \tan\theta}{1 - \tan\theta} \\
&= \frac{\left(1 + 2\tan\theta + \tan^2\theta\right) + \left(1 - 2\tan\theta + \tan^2\theta\right)}{1 - \tan^2\theta} \\
&= \frac{\left(1 + \tan^2\theta\right) + \left(1 + \tan^2\theta\right)}{1 - \tan^2\theta} \\
&= \frac{\sec^2\theta + \sec^2\theta}{1 - \tan^2\theta} \\
&= \frac{2\sec^2\theta}{1 - \tan^2\theta} \\
&= \text{R.S.}
\end{aligned}$$

The left side is equal to the right side, so the identity is proved.

PTS: 1 DIF: Moderate REF: 7.5 Sum and Difference Identities

LOC: 12.T6 TOP: Trigonometry

KEY: Procedural Knowledge | Conceptual Understanding | Communication

42. ANS:

$$\begin{aligned}
 \text{L.S.} &= \frac{\cot \theta + \csc \theta + \cos \theta}{\cot \theta} - \sin \theta \\
 &= (\cot \theta + \csc \theta + \cos \theta) \tan \theta - \sin \theta \\
 &= \cot \theta \tan \theta + \csc \theta \tan \theta + \cos \theta \tan \theta - \sin \theta \\
 &= 1 + \left(\frac{1}{\sin \theta} \right) \left(\frac{\sin \theta}{\cos \theta} \right) + (\cos \theta) \left(\frac{\sin \theta}{\cos \theta} \right) - \sin \theta \\
 &= 1 + \frac{1}{\cos \theta} + \sin \theta - \sin \theta \\
 &= 1 + \frac{1}{\cos \theta}
 \end{aligned}$$

$$\begin{aligned}
 \text{R.S.} &= \csc \theta (\sin \theta + \tan \theta) \\
 &= \csc \theta \sin \theta + \csc \theta \tan \theta \\
 &= \left(\frac{1}{\sin \theta} \right) (\sin \theta) + \left(\frac{1}{\sin \theta} \right) \left(\frac{\sin \theta}{\cos \theta} \right) \\
 &= 1 + \frac{1}{\cos \theta}
 \end{aligned}$$

The left and right sides simplify to the same expression, so the identity is proved.

PTS: 1

DIF: Difficult

REF: 7.3 Reciprocal and Quotient Identities

LOC: 12.T6

TOP: Trigonometry

KEY: Procedural Knowledge | Conceptual Understanding | Communication | Problem-Solving Skills

43. ANS:

$$\begin{aligned}
 \text{L.S.} &= \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} \\
 &= \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} \cdot \frac{\cos \theta + \sin \theta}{\cos \theta + \sin \theta} \\
 &= \frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta + 2 \sin \theta \cos \theta + \sin^2 \theta} \\
 &= \frac{\cos 2\theta}{(\cos^2 \theta + \sin^2 \theta) + \sin 2\theta} \\
 &= \frac{\cos 2\theta}{1 + \sin 2\theta} \\
 &= \text{R.S.}
 \end{aligned}$$

The left side is equal to the right side, so the identity is proved.

PTS: 1 DIF: Moderate REF: 7.6 Double-Angle Identities

LOC: 12.T6 TOP: Trigonometry

KEY: Procedural Knowledge | Conceptual Understanding | Communication | Problem-Solving Skills

44. ANS:

$$\begin{aligned}
 \text{L.S.} &= \frac{\cos \theta \cos 2\theta}{1 + \cos 2\theta} & \text{R.S.} &= \frac{\sin \theta (1 + \cos 4\theta)}{\sin 4\theta} \\
 &= \frac{\cos \theta \cos 2\theta}{1 + 2 \cos^2 \theta - 1} & &= \frac{\sin \theta (1 + 2 \cos^2 2\theta - 1)}{2 \sin 2\theta \cos 2\theta} \\
 &= \frac{\cos \theta \cos 2\theta}{2 \cos^2 \theta} & &= \frac{\sin \theta (2 \cos^2 2\theta)}{2 \sin 2\theta \cos 2\theta} \\
 &= \frac{\cos 2\theta}{2 \cos \theta} & &= \frac{\sin \theta \cos 2\theta}{\sin 2\theta} \\
 & & &= \frac{\sin \theta \cos 2\theta}{2 \sin \theta \cos \theta} \\
 & & &= \frac{\cos 2\theta}{2 \cos \theta}
 \end{aligned}$$

The left and right sides simplify to the same expression, so the identity is proved.

PTS: 1 DIF: Difficult REF: 7.6 Double-Angle Identities

LOC: 12.T6 TOP: Trigonometry

KEY: Procedural Knowledge | Conceptual Understanding | Communication | Problem-Solving Skills