PreCalc 12 Chapter 8 Review 2017 v1
Answer Section

MULTIPLE CHOICE

1. ANS: A
These are independent events, use FCP. $choice^{events}$

PTS: 1 DIF: Easy REF: 8.1 The Fundamental Counting Principle
LOC: 12.PCB1 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge

2. ANS: D
These are independent events, use FCP. $choice1 \cdot choice2$

PTS: 1 DIF: Easy REF: 8.1 The Fundamental Counting Principle
LOC: 12.PCB1 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge

3. ANS: B
These are independent events, use FCP. $choice1 \cdot choice2 \cdot choice3 \cdot choice4$

PTS: 1 DIF: Easy REF: 8.1 The Fundamental Counting Principle
LOC: 12.PCB1 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge

4. ANS: C
These are dependent events and order matters. Use permutations.

PTS: 1 DIF: Easy REF: 8.1 The Fundamental Counting Principle
LOC: 12.PCB1 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge

5. ANS: B
This is a permutation with repetition or a combination.

PTS: 1 DIF: Easy REF: 8.3 Permutations Involving Identical Objects
LOC: 12.PCB2 TOP: Permutations, Combinations and Binomial Theorem
KEY: Procedural Knowledge

6. ANS: B
This is a permutation with repetition.

PTS: 1 DIF: Easy REF: 8.3 Permutations Involving Identical Objects
LOC: 12.PCB2 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding

7. ANS: B PTS: 1 DIF: Easy
REF: 8.3 Permutations Involving Identical Objects LOC: 12.PCB2
TOP: Permutations, Combinations and Binomial Theorem KEY: Conceptual Understanding

8. ANS: D
Recall that you need to subtract 1 from the ordinals to get *n* and *r*.

PTS: 1 DIF: Easy REF: 8.5 Pascal's Triangle
LOC: 12.PCB4 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding

9. ANS: D
Recall that you need to add 1 to *n* and *r* to get the ordinals.

PTS: 1 DIF: Easy REF: 8.5 Pascal's Triangle
LOC: 12.PCB4 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding

10. ANS: B
Recall that *n* and *r* are part of the formula.

PTS: 1 DIF: Easy REF: 8.5 Pascal's Triangle
LOC: 12.PCB4 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding

11. ANS: D
These are dependent events and order matters. Use permutations.

PTS: 1 DIF: Moderate REF: 8.1 The Fundamental Counting Principle
LOC: 12.PCB1 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge

12. ANS: D
These are independent events, use FCP. $choice1 \cdot choice2 \cdot choice3 \cdot choice4$

PTS: 1 DIF: Moderate REF: 8.1 The Fundamental Counting Principle
LOC: 12.PCB1 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge

13. ANS: D
This is a permutation because order matters.

PTS: 1 DIF: Moderate REF: 8.2 Permutations of Different Objects
LOC: 12.PCB2 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge

14. ANS: C
Use a polynomial to solve. Or just guess and test.

PTS: 1 DIF: Moderate REF: 8.2 Permutations of Different Objects
LOC: 12.PCB2 TOP: Permutations, Combinations and Binomial Theorem
KEY: Procedural Knowledge

15. ANS: C
This is a permutation with repetition.

PTS: 1 DIF: Moderate REF: 8.3 Permutations Involving Identical Objects
LOC: 12.PCB2 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge

16. ANS: D
This is a permutation with repetition.

PTS: 1 DIF: Moderate REF: 8.3 Permutations Involving Identical Objects
LOC: 12.PCB2 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge

17. ANS: A
This is a combination because order doesn't matter.

PTS: 1 DIF: Moderate REF: 8.4 Combinations
LOC: 12.PCB3 TOP: Permutations, Combinations and Binomial Theorem
KEY: Procedural Knowledge | Conceptual Understanding

18. ANS: B
This is a combination because order doesn't matter.

PTS: 1 DIF: Moderate REF: 8.4 Combinations
LOC: 12.PCB3 TOP: Permutations, Combinations and Binomial Theorem
KEY: Procedural Knowledge | Conceptual Understanding

19. ANS: A
This is a combination because order doesn't matter.

PTS: 1 DIF: Moderate REF: 8.4 Combinations
LOC: 12.PCB3 TOP: Permutations, Combinations and Binomial Theorem
KEY: Procedural Knowledge | Conceptual Understanding

20. ANS: B
This is a combination because order doesn't matter. A line is a combination of 2 points.

PTS: 1 DIF: Moderate REF: 8.4 Combinations
LOC: 12.PCB3 TOP: Permutations, Combinations and Binomial Theorem
KEY: Procedural Knowledge | Conceptual Understanding

21. ANS: D
Make sure you put brackets around a and b for the powers.

PTS: 1 DIF: Moderate REF: 8.6 The Binomial Theorem
LOC: 12.PCB4 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge

22. ANS: B
Make sure you put brackets around a and b for the powers.

PTS: 1 DIF: Moderate REF: 8.6 The Binomial Theorem
LOC: 12.PCB4 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge

23. ANS: D
Make sure you put brackets around a and b for the powers.
The k^{th} term is: $\binom{n}{k-1}a^{n-(k-1)}b^{k-1}$. The exponents must add to n.

PTS: 1 DIF: Moderate REF: 8.6 The Binomial Theorem
LOC: 12.PCB4 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge

24. ANS: A
Make sure you put brackets around a and b for the powers.
The k^{th} term is: $\binom{n}{k-1}a^{n-(k-1)}b^{k-1}$. The exponents must add to n.

PTS: 1 DIF: Moderate REF: 8.6 The Binomial Theorem
LOC: 12.PCB4 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge

SHORT ANSWER

25. ANS:
-.5 for off by 1 errors
-.5 for incorrect concept
This is a permutation because order matters.
They can be seated in 336 different ways.

PTS: 1 DIF: Moderate REF: 8.2 Permutations of Different Objects
LOC: 12.PCB2 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge

26. ANS:
-.5 for off by 1 errors
-.5 for incorrect concept
This is a permutation because order matters.
There are 121 080 960 different ways.

PTS: 1 DIF: Moderate REF: 8.2 Permutations of Different Objects
LOC: 12.PCB2 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge

27. ANS:
-.5 for off by 1 errors
-.5 for incorrect concept
This is a permutation with repetition.
34 650 numbers

PTS: 1 DIF: Moderate REF: 8.3 Permutations Involving Identical Objects
LOC: 12.PCB2 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge

28. ANS:
-.5 for off by 1 errors
-.5 for incorrect concept
This is a permutation with repetition.
6930 ways

PTS: 1 DIF: Moderate REF: 8.3 Permutations Involving Identical Objects
LOC: 12.PCB2 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge

29. ANS:
Use a polynomial to solve. Or just guess and test.
$n = 8$

PTS: 1 DIF: Moderate REF: 8.4 Combinations
LOC: 12.PCB3 TOP: Permutations, Combinations and Binomial Theorem
KEY: Procedural Knowledge

30. ANS:
-.5 for off by 1 errors
-.5 for incorrect concept
This is a combination because order doesn’t matter. It is also FCP, so combination1*combination2
25 410 selections

PTS: 1 DIF: Moderate REF: 8.4 Combinations
LOC: 12.PCB3 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge

31. ANS:
-.5 for off by 1 errors
-.5 for incorrect concept
This is a combination because order doesn’t matter. It is also FCP, so combination1*combination2
756 ways

PTS: 1 DIF: Moderate REF: 8.4 Combinations
LOC: 12.PCB3 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge

32. ANS:
-.5 for off by 1 errors
-.5 for incorrect concept
This is a combination because order doesn't matter. It is also FCP, so combination1*combination2
100 selections

PTS: 1 DIF: Moderate REF: 8.4 Combinations
LOC: 12.PCB3 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge

33. ANS:

-.5 for off by 1 errors
$-32x^5 + 240x^4 - 720x^3 + 1080x^2 - 810x + 243$

PTS: 1 DIF: Moderate REF: 8.6 The Binomial Theorem
LOC: 12.PCB4 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge

34. ANS:

-.5 for off by 1 errors
$-3125x^{15} + 6250x^{12}y^2 - 5000x^9y^4 + 2000x^6y^6 - 400x^3y^8 + 32y^{10}$

PTS: 1 DIF: Moderate REF: 8.6 The Binomial Theorem
LOC: 12.PCB4 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge

35. ANS:
-.5 for off by 1 errors
The 2nd term, $7a^6b^{18}$

PTS: 1 DIF: Moderate REF: 8.6 The Binomial Theorem
LOC: 12.PCB4 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge | Problem-Solving Skills

36. ANS:
-.5 for off by 1 errors
-.5 for incorrect concept
This is a permutation because order matters. It is also FCP, so $permutation1 \cdot permutation2$
There are 4950 possible arrangements.

PTS: 1 DIF: Difficult REF: 8.2 Permutations of Different Objects
LOC: 12.PCB2 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge

PROBLEM

37. ANS:
There are 13 hearts and 26 black cards in a deck of 52 cards.
So, there are ${}_{13}C_3$ ways to deal 3 hearts, and ${}_{26}C_4$ ways to deal 4 black cards.
Use the fundamental counting principle.

$$\left({}_{13}C_3\right)\left({}_{26}C_4\right) = \left(\frac{13!}{(13-3)!3!}\right)\left(\frac{26!}{(26-4)!4!}\right)$$

$$= 4\,275\,700$$

So, 4 275 700 hands of 3 hearts and 4 black cards can be dealt.

PTS: 1 DIF: Moderate REF: 8.4 Combinations
LOC: 12.PCB3 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge | Communication | Problem-Solving Skills

38. ANS:
Use the binomial theorem.

$$\left(x+y\right)^n = {}_nC_0x^n + {}_nC_1x^{n-1}y + \ldots + {}_nC_ny^n$$

Substitute: $n = 5, x = \frac{3}{2}x, y = \frac{4}{5}y$

$$\left(\frac{3}{2}x+\frac{4}{5}y\right)^5 = {}_5C_0\left(\frac{3}{2}x\right)^5 + {}_5C_1\left(\frac{3}{2}x\right)^4\left(\frac{4}{5}y\right) + {}_5C_2\left(\frac{3}{2}x\right)^3\left(\frac{4}{5}y\right)^2$$

$$+ {}_5C_3\left(\frac{3}{2}x\right)^2\left(\frac{4}{5}y\right)^3 + {}_5C_4\left(\frac{3}{2}x\right)\left(\frac{4}{5}y\right)^4 + {}_5C_5\left(\frac{4}{5}y\right)^5$$

$$= \frac{243}{32}x^5 + \frac{81}{4}x^4y + \frac{108}{5}x^3y^2 + \frac{288}{25}x^2y^3 + \frac{384}{125}xy^4 + \frac{1024}{3125}y^5$$

PTS: 1 DIF: Moderate REF: 8.6 The Binomial Theorem
LOC: 12.PCB4 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge | Communication

39. ANS:
The first term is $32x^5$. The exponent of x is 5, and x is the only variable in the expansion.
So, the binomial has the form $(ax+b)^5$.
The coefficient a must be $\sqrt[5]{32}$: $a = 2$
The last term is 243, so b must be $\sqrt[5]{243}$: $b = 3$

The binomial power is $\left(2x+3\right)^5$.

PTS: 1 DIF: Moderate REF: 8.6 The Binomial Theorem
LOC: 12.PCB4 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge | Communication | Problem-Solving Skills

40. ANS:
Use the fundamental counting principle.
There are 8 consecutive seats.
Anyone of the 8 people can sit in the first seat.
Her or his partner must sit in the second seat.
That leaves any one of 6 people to sit in the third seat.
Her or his partner must sit in the fourth seat.
That leaves any one of 4 people to sit in the fifth seat.
Her or his partner must sit in the sixth seat.
Continue the pattern for all 8 seats.
So, the number of seating arrangements is: $(8)(1)(6)(1)(4)(1)(2)(1) = 384$
There are 384 possible seating arrangements.

PTS: 1 DIF: Difficult REF: 8.1 The Fundamental Counting Principle
LOC: 12.PCB1 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge | Communication | Problem-Solving Skills

41. ANS:
Since the wreath is circular, the same flower can always be chosen to be first, no matter the order of the flowers on the wreath. So, consider one flower to be fixed, then consider the arrangements of the remaining 8 flowers.
The number of ways of arranging 8 flowers is: $8! = 40\,320$
There are 40 320 ways to arrange 9 flowers on the wreath.

PTS: 1 DIF: Difficult REF: 8.2 Permutations of Different Objects
LOC: 12.PCB2 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge | Communication | Problem-Solving Skills

42. ANS:
There are 2 vowels. The number of ways the vowels can be arranged is: 2!
There are 5 consonants.
Since the vowels have to be together, consider them as 1 object.
So, there are 6 objects: the vowels and 5 consonants.
2 pairs of the consonants are identical.
So, the number of permutations of the 6 objects is: $\frac{6!}{2!2!}$.
Use the fundamental counting principle:

$$\left(\frac{6!}{2!2!}\right)(2!) = 360$$

So, all the letters in RHUBARB can be arranged in 360 ways with the vowels together.

PTS: 1 DIF: Difficult REF: 8.3 Permutations Involving Identical Objects
LOC: 12.PCB2 TOP: Permutations, Combinations and Binomial Theorem
KEY: Procedural Knowledge | Conceptual Understanding | Communication | Problem-Solving Skills

43. ANS:
The club has 29 members, 15 of whom are girls.
So, 14 of the members must be boys.
The 5-member committee must have at least 3 girls.
So, the committee could have 3 girls, 4 girls, or 5 girls.
If the committee has 3 girls, then it must have 2 boys.
Number of ways of choosing a committee of 3 girls and 2 boys is:
$\left({}_{15}C_3\right)\left({}_{14}C_2\right)$
If the committee has 4 girls, then it must have 1 boy.
Number of ways of choosing a committee of 4 girls and 1 boy is:
$\left({}_{15}C_4\right)\left({}_{14}C_1\right)$
If the committee has 5 girls, then it must have no boys.
Number of ways of choosing a committee of 5 girls and 0 boys is:
$\left({}_{15}C_5\right)\left({}_{14}C_0\right)$
So, the total number of possible committees is:
$\left({}_{15}C_3\right)\left({}_{14}C_2\right)+\left({}_{15}C_4\right)\left({}_{14}C_1\right)+\left({}_{15}C_5\right)\left({}_{14}C_0\right)$
$=\left(\frac{15!}{(15-3)!3!}\right)\left(\frac{14!}{(14-2)!2!}\right)+\left(\frac{15!}{(15-4)!4!}\right)\left(\frac{14!}{(14-1)!1!}\right)+\left(\frac{15!}{(15-5)!5!}\right)\left(\frac{14!}{(14-0)!0!}\right)$
= 63 518
There are 63 518 possible committees.

PTS: 1 DIF: Difficult REF: 8.4 Combinations
LOC: 12.PCB3 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Procedural Knowledge | Communication | Problem-Solving Skills

44. ANS:
In each row after the 1st row, any number that is not equal to 1 is the sum of the two numbers above it.
For example, the second number in a row is the sum of the first and second numbers in the previous row. The third number in a row is the sum of the second and third numbers in the previous row.
So, the $(r+1)$th number in a row is the sum of the rth and $(r+1)$th numbers in the previous row. For the row numbered $(n+2)$, the previous row is numbered $(n+1)$.

So, the $(r+1)$th number in row $(n+2)$ is the sum of the rth and $(r+1)$th numbers in row $(n+1)$.

The rth number in row $(n+1)$ is ${}_nC_{r-1}$ and the $(r+1)$th number in row $(n+1)$ is ${}_nC_r$.

So, ${}_{n+1}C_r = {}_nC_{r-1} + {}_nC_r$, where ${}_{n+1}C_r$ is the $(r+1)$th number in row $(n+2)$.

PTS: 1 DIF: Difficult REF: 8.5 Pascal's Triangle
LOC: 12.PCB4 TOP: Permutations, Combinations and Binomial Theorem
KEY: Conceptual Understanding | Communication | Problem-Solving Skills