PreCalc 12 Chp 2 Review/Ref Sheet

Note Title 2013-09-29

Properties of Radical Functions

Domain: $x \geq 0$ } for $\sqrt{x}$
Range: $y \geq 0$ }

Invariant points occur where $f(x) = \sqrt{f(x)}$, ie $f(x) = 0$ or $f(x) = 1$.

Radical of Quadratic Functions

$y = \sqrt{f(x)}$ where $f(x)$ is quadratic.

Domain: $f(x) \geq 0$ Range: it depends.

Sketching: Use domain, range, & invariant points. You can also calculate additional table of values. Then draw a smooth curve.

3 cases: no x-intercepts, inner interval, outer intervals.

If $0 < f(x) < 1$, plot $\sqrt{f(x)}$ above $f(x)$.
If $f(x) > 1$, plot $\sqrt{f(x)}$ below $f(x)$.

For $\sqrt[3]{x}$, invariant points occur at $f(x) = -1, 0,$ or 1.

Domain: $x \in \mathbb{R}$, Range: it depends.

If $-1 < f(x) < 1$, plot $\sqrt[3]{f(x)}$ below $f(x)$.
If $f(x) < -1$, plot $\sqrt[3]{f(x)}$ above $f(x)$.

Analyzing Rational Functions. $\frac{P(x)}{Q(x)}$

domain? $x \in \mathbb{R}$ except NPV's

Where are the vertical asymptotes or holes?

At the zeroes of $Q(x)$

When do you have holes vs. V.A.?

You get holes where you cancel out a factor with $P(x)$, otherwise it's a V.A.

When do you get a H.A.?

When the degree of $Q(x)$ is $\geq$ degree $P(x)$.

If $\deg(Q(x)) = \deg(P(x))$ then H.A. $y = a/b$ where 'a' is the L.C. of $P(x)$ and 'b' is the L.C. of $Q(x)$.

Otherwise H.A. is $y = 0$.

When do you get a S.A. (slant or oblique asymptote)

When $\deg(P(x)) = \deg(Q(x)) + 1$ and there is a V.A.

You can calculate the S.A. by polynomial division.

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1. Use NPV's to find V.A.'s and holes.
2. H.A.'s occur when $\deg(P(x)) \leq \deg(Q(x))$
3. S.A.'s occur when $\deg(P(x)) = \deg(Q(x)) + 1$ & $\nexists$ V.A.
   Use polynomial division for S.A.
4. Find behaviour near V.A.'s (generally $\pm .1$ or $\pm .01$ away).
5. Find behaviour near H.A.'s (generally $x = \pm 100$ or ± 1000)
6. Find x-intercepts (the zeroes of $P(x)$).
7. Find the y-intercept (set $x=0$ and solve for y)
8. Label all intercepts.
9. Join points smoothly.