PreCalc 12 Chp 3 Review/Reference Sheet

Note Title 2013-09-29

Transformations

$y = af(b(x-h)) + k$ is a transform of $f(x)$.

The graph before the transform is called the preimage.
The graph of the transform is called the image.
If $f(bx-c)$, you must factor: $h = c/b$

$h > 0$ right, $h < 0$ left
$k > 0$ up, $k < 0$ down
$|a| > 1$ stretch, $|a| < 1$ compress, $a < 0$ reflect on x-axis
$|b| > 1$ compress, $|b| < 1$ stretch, $b < 0$ reflect on y-axis

Domain:
- a – reflect range on x-axis if $a < 0$
- b – reflect domain on y-axis if $b < 0$
- h – will shift domain
- k – will shift range

If $f(x) = f(-x)$, f is an even function
If $-f(x) = f(-x)$ [or $f(x) = -f(-x)$], f is an odd function
Use substitution to test if a fn is even or odd.

Odd degree polynomials can never be even fns.
Even degree " " " " odd fns.
Odd functions must have opposite roots and zero as a root: eg) $0, \pm 1, \pm 4$
Even functions must have opposite roots: eg) $\pm 2, \pm 5$

Sketching: If (x, y) is a point of the preimage, then the point of the image is $(x/b + h, ay + k)$.
If (x, y) is a point of the image, then the point of the preimage is $(b(x-h), (y-k)/a)$

eg)

11. A transformation image of the graph of $y = f(x)$ is represented by the equation $y - 1 = -2f\left(\frac{x+5}{3}\right)$. The point $(7, 5)$ lies on the image graph. What are the coordinates of the corresponding point on the graph of $y = f(x)$?

$k = 1$ $a = -2$ $b = 1/3$ $h = -5$

$(x_i, y_i) = (7, 5)$ $\quad (x_p, y_p) = \left(b(x_i - h), (y_i - k)/a \right)$

$= \left(\frac{1}{3}(7 - (-5)), (5 - 1)/-2 \right)$

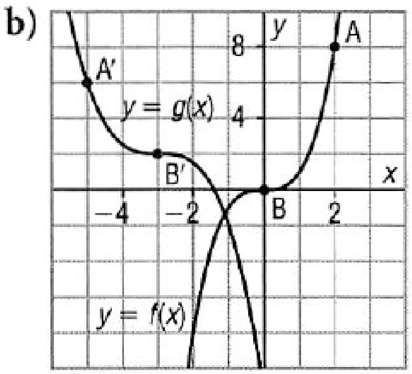
$= (4, -2)$

Determining a Transform Function.

One point on a preimage and an image can determine one of 'a' or 'k' and one of 'b' or 'h'.

Two points of a preimage and an image can determine the full transform. Use system of linear eqns.

eg)



$A_x = b(A'_x - h) \Rightarrow 2 = b(-5 - h)$

$B_x = b(B'_x - h) \Rightarrow 0 = b(-3 - h)$

$\Rightarrow 2 = -5b - bh$

$\Rightarrow 0 = -3b - bh$

$2 = -2b \Rightarrow b = -1$

$0 = -3(-1) - (-1)h$

$-3 = h$

$A'_y = a A_y + k \Rightarrow 6 = a8 + k$

$B'_y = a B_y + k \Rightarrow 2 = a0 + k \Rightarrow k = 2$

$\Rightarrow 6 = 8a + 2 \Rightarrow 8a = 4 \Rightarrow a = \frac{1}{2}$

Transform Function: $\boxed{y = \frac{1}{2} f(-(x+3)) + 2}$

Inverse Relations (as opposed to functions)

If given a graph, reflect curve on $y = x$. Can also take key points; swap x & y; plot; then mirror curve between the key points.

In general: swap domain and range for the inverse.

To Determine the Inverse Algebraically:

- swap all x's with y's and y's with x's.
- solve for y.

eg) $y = 3(x+2)^2 - 5$

$3(y+2)^2 - 5 = x$

$3(y+2)^2 = x + 5$

$(y+2)^2 = \frac{x+5}{3}$

$y + 2 = \pm\sqrt{\frac{x+5}{3}}$

$\boxed{y = -2 \pm \sqrt{\frac{x+5}{3}}}$