Transformations

\[ y = af(b(x-h)) + k \]

is a transform of \( f(x) \).

The graph before the transform is called the preimage.
The graph of the transform is called the image.

If \( f(\text{bx-c}) \), you must factor:
- \( h > 0 \) right, \( h < 0 \) left
- \( k > 0 \) up, \( k < 0 \) down
- \( |a| > 1 \) stretch, \( |a| < 1 \) compress, \( a < 0 \) reflect on x-axis
- \( |b| > 1 \) compress, \( |b| < 1 \) stretch, \( b < 0 \) reflect on y-axis

Domain:  
- \( a \) - reflect range on x-axis if \( a < 0 \)
- \( b \) - reflect domain on y-axis if \( b < 0 \)
- \( h \) - will shift domain
- \( k \) - will shift range

If \( f(x) = f(-x) \), \( f \) is an even function.
If \( -f(x) = f(-x) \) or \( f(x) = -f(-x) \), \( f \) is an odd function.

Use substitution to test if a fn is even or odd.

Odd degree polynomials can never be even fns.
Even degree polynomials can never be odd fns.
Odd functions must have opposite roots and zero as a root:  \( \text{eg) } 0, \pm 1, \pm 4 \)
Even functions must have opposite roots:  \( \text{eg) } \pm 2, \pm 5 \)

Sketching:  If \((x, y)\) is a point of the preimage, then the point of the image is \( (\frac{x}{b} + h, ay + k) \).
If \((x, y)\) is a point of the image, then the point of the preimage is \( (b(x-h), (y-k)/a) \).

\( \text{eg) } \) A transformation image of the graph of \( y = f(x) \) is represented by the equation \( y - 1 = -2f(\frac{x + 5}{3}) \). The point \((7, 5)\) lies on the image graph. What are the coordinates of the corresponding point on the graph of \( y = f(x) \)?

\( k = 1 \)  \( a = -2 \)  \( b = \frac{1}{3} \)  \( h = -5 \)
\[(x_i, y_i) = (7, 5) \quad \begin{align*}
(x'_i, y'_i) &= b(x_i - h), \quad (y_i - k)/a \\
&= \left(\sqrt{3}(7-(-5)), \frac{(5-1)}{-2}\right) \\
&= (4, -2)
\end{align*}\]

**Determining a Transform Function.**

One point on a preimage and an image can determine one of 'a' or 'k' and one of 'b' or 'h'.

Two points of a preimage and an image can determine the full transform. Use system of linear eqns.

**eg)**

\[
\begin{align*}
A_x &= b(A'_x - h) \quad \Rightarrow \quad 2 = b\left(-5 - h\right) \\
B_x &= b(B'_x - h) \quad \Rightarrow \quad 0 = b\left(-3 - h\right) \\
\Rightarrow \quad 2 &= -5b - bh \\
\Rightarrow \quad 0 &= -3b - bh \\
\Rightarrow \quad 2 &= -2b \quad \Rightarrow \quad b = -1 \\
\Rightarrow \quad 0 &= -3(-1) - bh \quad \Rightarrow \quad -3 = h \\
\end{align*}
\]

\[
A'_y = aA_y + k \quad \Rightarrow \quad 6 = aB + k \\
B'_y = aB_y + k \quad \Rightarrow \quad 2 = a0 + k \quad \Rightarrow \quad k = 2 \\
\Rightarrow \quad 6 &= 8a + 2 \quad \Rightarrow \quad 8a = 4 \quad \Rightarrow \quad a = \frac{1}{2}
\]

**Transform Function:** \(y = \frac{1}{2} f(-x+3) + 2\)

**Inverse Relations (as opposed to functions).**

If given a graph, reflect curve on \(y=x\). Can also take key points; swap \(x\) & \(y\); plot; then mirror curve between the key points.

In general: swap domain and range for the inverse.

To Determine the Inverse Algebraically:

- Swap all \(x\)'s with \(y\)'s and \(y\)'s with \(x\)'s.
- Solve for \(y\).

**eg)**

\[
\begin{align*}
y &= 3(x + 2)^2 - 5 \\
3(y + 2)^2 - 5 &= x \\
3(y + 2)^2 &= x + 5 \\
(y + 2)^2 &= \frac{x+5}{3} \\
y + 2 &= \sqrt{\frac{x+5}{3}} \\
y = -2 \pm \sqrt{\frac{x+5}{3}}
\end{align*}
\]