PreCalc 12 Chp 6 Review

Note Title 2013-09-29

Trigonometric Ratios in Standard Position.

Angles in Standard Position start with the initial side along the positive x-axis. It rotates counter-clockwise until the terminal side is reached. (positive dir)
Every time we go all the way around the circle CCW, we add $360°$ (2π) to the angle.
Every time we go all the way around the circle CW, we subtract $360°$ (2π) from the angle.
Any time two angles have the same terminal side, we say they are coterminal.

Equation of Unit Circle: $x^2+y^2=1$

$\sin\theta = y$ $\quad\cos\theta = x$ $\quad\tan\theta = y/x$

angle ratio never $\tan(y/x)$

If we generalize to any circle with radius: r

$x^2+y^2=r^2$ $\quad\sin\theta = \frac{y}{r}$ $\quad\cos\theta = \frac{x}{r}$ $\quad\tan\theta = \frac{y}{x}$

This gives us the mnemonic: syr, cxr, tyx
sir, kicks'r, ticks

Reciprocal Trig Ratios.

$\frac{1}{\sin\theta} = \frac{r}{y} = \csc\theta$ cosecant

$\frac{1}{\cos\theta} = \frac{r}{x} = \sec\theta$ secant

$\frac{1}{\tan\theta} = \frac{x}{y} = \cot\theta$ cotangent

The reference angle (θ') is the smallest positive angle that has one side along the x-axis and the other side on the terminal side.

Q-I: $\theta' = \theta$		$\theta = \theta'$	
Q-II: $\theta' = 180-\theta$	or $\pi-\theta$	$\theta = 180-\theta'$	or $\pi-\theta'$
Q-III: $\theta' = \theta-180$	or $\theta-\pi$	$\theta = 180+\theta'$	or $\pi+\theta'$
Q-IV: $\theta' = 360-\theta$	or $2\pi-\theta$	$\theta = 360-\theta'$	or $2\pi-\theta'$

Sin	All
Tan	Cos

All Students Take Calculus.
(positive ratios)

Angles in Standard Position and Arc Length

Definition: Radian is the ratio between the arc length and the radius.

Properties (for radians):

$\theta = \dfrac{s}{r}$ $s = |r\theta|$ $r = \dfrac{s}{\theta}$

radius

arclength

subtended angle

Converting between degrees and radians

$deg = \dfrac{180\, rads}{\pi}$ $rad = \dfrac{\pi\, deg}{180}$

Special Angles:

$0° + 90°n$	$0 + \frac{\pi}{2}n$
$30° + 90°n$	$\frac{\pi}{6} + \frac{\pi}{2}n$
$45° + 90°n$	$\frac{\pi}{4} + \frac{\pi}{2}n$
$60° + 90°n, \; n \in \mathbb{Z}$	$\frac{\pi}{3} + \frac{\pi}{2}n$

Definition Principal Angle: smallest positive coterminal angle. ie $0 \le \theta < 2\pi$ or $0° \le \theta < 360°$

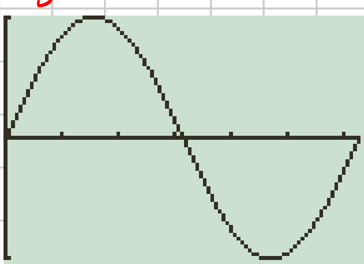
Area of a Sector:

$A = \dfrac{\theta}{2\pi}(\pi r^2)$ angle must be in radians

$\dfrac{\theta}{360°}(\pi r^2)$ for deg

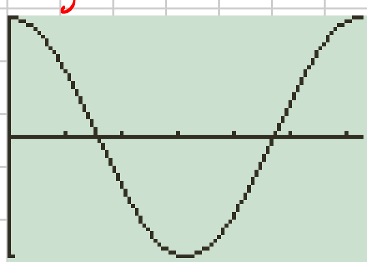
Graphing Trig Functions

$y = \sin x$



← Period $= 2\pi$ →

$y = \cos x$



← Period $= 2\pi$ →

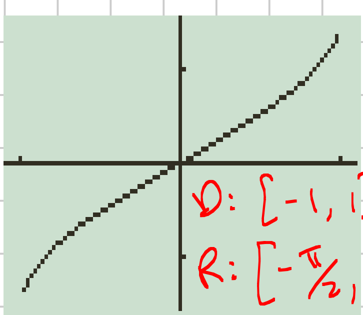
$y = \tan x$

← → Period $= \pi$

V.A.

NPV. So not in domain.

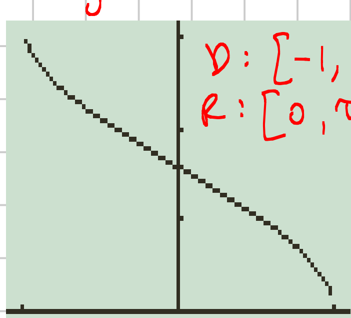
$y = \sin^{-1} x$



D: $[-1, 1]$

R: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

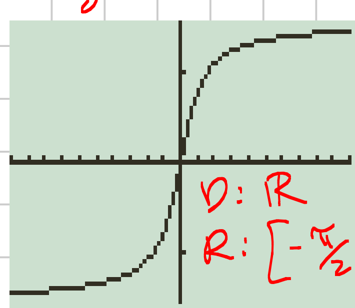
$y = \cos^{-1} x$



D: $[-1, 1]$

R: $[0, \pi]$

$y = \tan^{-1} x$



D: $\mathbb{R}$

R: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

sin & cos $\frac{2\pi}{b}$ or tan $\frac{\pi}{b}$

Period - The length for a function to repeat.
Centerline - (max + min)/2
Amplitude - (max - min)/2 or max - centerline
Phase Shift - AKA horizontal shift
Sinusoidal Functions - sine and cosine

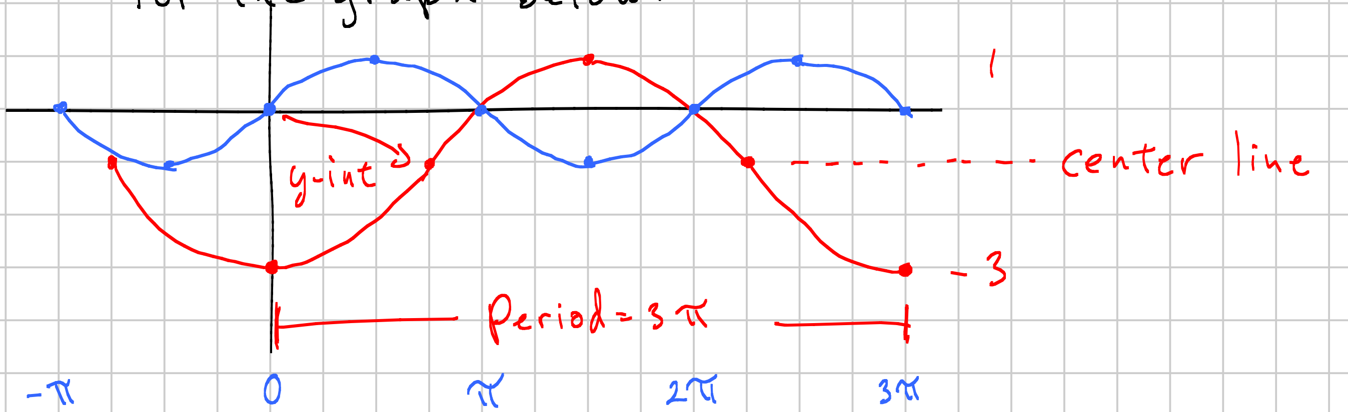
General Form: $a\,f(b(x-c))+d$

amplitude, h comp/stretch, phase shift, centerline

Easy to determine a single transform.

Use steps to find combo of transforms.
1 - Calculate the center line.
2 - Calculate the amplitude.
3 - Calculate the horizontal scaling.
4 - Calculate the phase shift. from y-int (0 - sin, 1 - cos)

eg) Determine the constants a, b, c, & d in $y = a\sin(b(x-c)) + d$ and write the function for the graph below:



1: $d = [1 + (-3)]/2 = -2/2 = -1$
2: $a = 1 - (-1) = 2$
3: $b = \frac{2\pi}{3\pi} = 2/3$
4: $c = 3\pi/4$

$y = 2\sin(2/3(x - 3\pi/4)) - 1$

For cosine: $y = 2\cos(2/3(x - 3\pi/2)) - 1$

Word Problems: Pick out key information to determine: a, b, c, d. Sometimes the info will be direct; other times indirect.

radius ≡ amplitude
diameter ≡ amplitude(2) ≡ peak to peak
revolution ≡ once around ≡ full circle ≡ full turns ≡ cycle
min ≡ low ≡ least
max ≡ high ≡ most
centerline ≡ mean ≡ average ≡ center ≡ middle