
Angles in Standard Position start with the initial side along the positive x-axis. It rotates counter-clockwise until the terminal side is reached. (positive dir)

Every time we go all the way around the circle CCW, we add $360^\circ$ ($2\pi$) to the angle.

Every time we go all the way around the circle CW, we subtract $360^\circ$ ($2\pi$) from the angle.

Any time two angles have the same terminal side, we say they are coterminal.

Equation of Unit Circle: $x^2 + y^2 = 1$

$\sin \theta = \frac{y}{r}$ \hspace{3cm} $\cos \theta = \frac{x}{r}$ \hspace{3cm} $\tan \theta = \frac{y}{x}$

If we generalize to any circle with radius: $r$

This gives us the mnemonic: Sine, Cosine, Tangent

Reciprocal Trig Ratios:

$\frac{1}{\sin \theta} = \csc \theta$ \hspace{3cm} $\frac{1}{\cos \theta} = \sec \theta$ \hspace{3cm} $\frac{1}{\tan \theta} = \cot \theta$

Cosecant \hspace{3cm} Secant \hspace{3cm} Cotangent

The reference angle ($\theta'$) is the smallest positive angle that has one side along the x-axis and the other side on the terminal side.

- Q-I: $\theta' = \theta$
- Q-II: $\theta' = 180 - \theta$ or $\pi - \theta$
- Q-III: $\theta' = \theta - 180$ or $\pi - \theta$
- Q-IV: $\theta' = 360 - \theta$ or $2\pi - \theta$

$\sin \theta$, $\csc \theta$, $\sec \theta$, $\cot \theta$ (positive ratios)

All Students Take Calculus.
Angles in Standard Position and Arc Length

Definition: A radian is the ratio between the arc length and the radius.

Properties (for radians):
\[ \theta = \frac{\text{arc length}}{\text{radius}} \]
\[ r = \frac{s}{\theta} \]

Converting between degrees and radians
\[ \text{deg} = \frac{180 \text{ rad}}{\pi} \quad \text{rad} = \frac{\pi \text{ deg}}{180} \]

Special Angles:
- \( 0^\circ + 90^\circ n \)
- \( 30^\circ + 90^\circ n \)
- \( 45^\circ + 90^\circ n \)
- \( 60^\circ + 90^\circ n \), \( n \in \mathbb{Z} \)

Definition Principal Angle: Smallest positive coterminal angle, i.e. \( 0 \leq \theta < 2\pi \) or \( 0^\circ \leq \theta < 360^\circ \)

Area of a Sector:
\[ A = \frac{\theta}{2\pi} \left( \pi r^2 \right) \]

Graphing Trig Functions
\[ y = \sin x \quad y = \cos x \quad y = \tan x \]

- Period = \( 2\pi \)
- Period = \( 2\pi \)
- Period = \( \pi \)

\[ y = \sin^{-1} x \quad y = \cos^{-1} x \quad y = \tan^{-1} x \]

- \( D: [-1, 1] \)
- \( R: [0, \pi] \)
- \( D: \mathbb{R} \)
- \( R: [-\frac{\pi}{2}, \frac{\pi}{2}] \)

V.A. No V. so not in domain.
Period - The length for a function to repeat.
Center line - \((\text{max} + \text{min})/2\)
Amplitude - \((\text{max} - \text{min})/2\) or \(\text{max} - \text{center line}\)
Phase Shift - AKA horizontal shift
Sinusoidal Functions - sine and cosine

**General Form:** \(a \sin (b(x-c)) + d\)

- **a** - Amplitude, \(h\) - comp/stretch
- **b** - Period, \(n\) - comp/stretch
- **c** - Phase shift, \(y\)-int
- **d** - Center line

Use steps to find combo of transforms:
1. Calculate the center line.
2. Calculate the amplitude.
3. Calculate the horizontal scaling.
4. Calculate the phase shift. From y-int

**Eq:** Determine the constants \(a, b, c, \& d\) in \(y = a \sin (b(x-c)) + d\) and write the function for the graph below:

1. \(d = [(1 + (-3))/2] = -2/2 = -1\)
2. \(a = 2(1 - (-1)) = 2\)
3. \(b = 2\pi / (3\pi - 2\pi / 3) = 2/3\)
4. \(c = 3\pi / 4\)

\(y = 2 \sin (2\pi/3 (x-3\pi/4)) - 1\)

For cosine: \(y = 2 \cos (2\pi/3 (x-3\pi/2)) - 1\)

**Word Problems:** Pick out key information to determine: \(a, b, c, \& d\). Sometimes the info will be direct; other times indirect.
Radius = amplitude
diameter = amplitude (2) = peak to peak
revolution = once around = full circle = full turns = cycle
min = low = least
max = high = most
Centerline = mean = average = center = middle