PreCalc 12 Chp 7 Review

Solving Trigonometric Equations Graphically

Unless an interval is given, then there are an infinite number of solutions.

Make sure that your calculator in the correct mode; ie. radians or degrees.

Solving Trigonometric Equations Algebraically

General Solutions -

tan — only 1 period = π/b

sin & cos

if $\sin bx = 1$ or $\cos bx = 1$ or $\cos bx = -1$ or $\sin bx = -1$

— only 1 sol period $2\pi/b$

if $\sin bx = 0$ or $\cos bx = 0$

— only 1 sol period π/b

if $\sin bx = \pm 1/\sqrt{2}$ or $\cos bx = \pm 1/\sqrt{2}$

— only 1 sol period $\pi/2b$

otherwise

- 2 solns period $2\pi/b$
- use ASTC to figure out the 2 quadrants
- 2 pos & 2 neg for each trig function in the 4 quadrants.

Use substitution to make factoring easier, eg. $y = \sin x$

Reciprocal and Quotient Identities.

$\csc\theta = \dfrac{1}{\sin\theta}$ $\qquad \sec\theta = \dfrac{1}{\cos\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\qquad \cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$

$\csc\theta\sin\theta = 1$ $\qquad \sec\theta\cos\theta = 1$ $\qquad \cot\theta\tan\theta = 1$

Simplify to no fractions

1. We can only cancel out factors if they are in the numerator & denominator of the same term.
2. We can only cancel out terms if there are no multiplies or divides between them.
3. We can never cancel factors that are in parameters.

Pythagorean Identities.

$\cos^2\theta + \sin^2\theta = 1$
$\sin^2\theta = 1 - \cos^2\theta$
$\cos^2\theta = 1 - \sin^2\theta$

$\tan^2\theta + 1 = \sec^2\theta$
$\tan^2\theta = \sec^2\theta - 1$
$\sec^2\theta - \tan^2\theta = 1$

$1 + \cot^2\theta = \csc^2\theta$
$\cot^2\theta = \csc^2\theta - 1$
$\csc^2\theta - \cot^2\theta = 1$

Sum and Difference Identities

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$
$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Double Angle Identities

$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$\sin 2\theta = 2\sin\theta\cos\theta$
$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$\cos 2\theta = 2\cos^2\theta - 1$
$\cos 2\theta + 1 = 2\cos^2\theta$

$\cos 2\theta = 1 - 2\sin^2\theta$
$2\sin^2\theta = 1 - \cos 2\theta$

Solving Exact Ratio Problems

- must use identities.

eg) $\dfrac{\tan 160^\circ + \tan 50^\circ}{1 - \tan 160^\circ \tan 50^\circ} = \tan(160^\circ + 50^\circ) = \tan 210^\circ$
$= -1/\sqrt{3}$

Common Error: $\tan 150^\circ + \tan 50^\circ \approx .8\cancel{2}778$

eg) θ in Q2, $\sin\theta = 5/13$, find $\cos 2\theta$

$\cos\theta = -12/13$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(-12/13\right)^2 - \left(5/13\right)^2 = \frac{144-25}{169} = \frac{119}{169}$

Common Error: $\cos 2\theta = \cos\left(2\left(\frac{-12}{13}\right)\right) = \cancel{-\frac{24}{13}}$

eg) $\tan\alpha = -24/7$, $\tan\beta = -35/12$, α & β in Q-IV, find $\cos(\alpha+\beta)$.

$\sin\alpha = -24/25$ $\quad \cos\alpha = 7/25$ $\quad \sin\beta = -35/37$ $\quad \cos\beta = 12/37$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \left(\frac{7}{25}\right)\left(\frac{12}{37}\right) - \left(\frac{-24}{25}\right)\left(\frac{-35}{37}\right)$

$= \frac{-756}{925}$

Common Error: $\cos(\alpha+\beta) = \cos\left(\frac{7}{25}\right) + \cos\left(\frac{12}{37}\right) = \cancel{\frac{559}{925}}$

Verify: This is substituting one or two values to check if the identity works.

Prove: This is using algebra to show that the identity works for all values. For proofs, you are only allowed to use algebra on one side of the identity. Must show all steps!

Strategy:
- Choose to work with more complex side.
- Look to other side for hints and identities
- Look to multiply for difference of squares or just plain squaring - the result is usually an identity.
- Convert all trig fns to sin & cos if stuck
- Simplify to single fraction.
- Make conversion to look like simpler side.

Difference between identity and equation is that the identity can't be solved; it will simply reduce to a truth, ie. $1 = 1$ or $1 = 0$.

Speed Tip: get used to recognizing the identities so that you can quickly substitute.

Find exact ratios from special angles.

I: $15^\circ = 45^\circ - 30^\circ$ $\qquad$ $75^\circ = 45^\circ + 30^\circ$

$\frac{\pi}{12} = \frac{\pi}{4} - \frac{\pi}{6}$ $\qquad$ $\frac{5\pi}{12} = \frac{\pi}{4} + \frac{\pi}{6}$

II: $105^\circ = 135^\circ - 30^\circ$ $\qquad$ $165^\circ = 135^\circ + 30^\circ$

$\frac{7\pi}{12} = \frac{3\pi}{4} - \frac{\pi}{6}$ $\qquad$ $\frac{11\pi}{12} = \frac{3\pi}{4} + \frac{\pi}{6}$

III: $195^\circ = 225^\circ - 30^\circ$ $\qquad$ $255^\circ = 225^\circ + 30^\circ$

$\frac{13\pi}{12} = \frac{5\pi}{4} - \frac{\pi}{6}$ $\qquad$ $\frac{17\pi}{12} = \frac{5\pi}{4} + \frac{\pi}{6}$

IV: $285^\circ = 315^\circ - 30^\circ$ $\qquad$ $345^\circ = 315^\circ + 30^\circ$

$\frac{19\pi}{12} = \frac{7\pi}{4} - \frac{\pi}{6}$ $\qquad$ $\frac{23\pi}{12} = \frac{7\pi}{4} + \frac{\pi}{6}$

$\cancel{90^\circ + 75^\circ}$

Solve. Okay to use calculator especially if it asks you to round answer.

Use algebra to solve. Use algebra until you need to find the angle, the use calculator for inverse trig. Again, especially if it asks you to round.

Find exact angle. Use algebra all the way through.