PreCalc 12 Chp 8 Review

Note Title 2017-01-03

The Fundamental Counting Principle $-(n_1)(n_2)(n_3)\ldots$
This applies to what we call independent events, ie. each event has no impact on the next event.

Tree Diagram - this can be used for small cases. Draw all nodes then count the leaves. Trees are drawn upside-down.

Pigeonhole Principle - If there are n items and m pigeonholes and $n > m$, then at least one pigeonhole contains more than one item.

Backwards problems - use roots or logs depending on whether you need the base or exponent.

Permutations of Different Objects.
To permute is to arrange. Permutation is to find all the different way to arrange objects, so order is important. This is a slight variation on FCP.

We can generalize for any n objects that the number of arrangements is: $(n)(n-1)(n-2)\cdots(3)(2)(1)$.
There is a function for this operation called factorial. The notation is $n!$ By definition, $0! = 1$

Backwards problem: use trial & error or find zeroes with graphing or factor property

Permutations Involving Identical Objects.
Divide by the factorial of the number of repetitions.
Be careful when deciding between this and FCP!

Combinations - used when order does not matter.

$${}_nC_r = \frac{n!}{(n-r)! \cdot r!} = \binom{n}{r}$$

This is similar to the treatment of identical objects in an arrangement.

If there are multiple combinations, we use the Fundamental Counting Principle and multiply the answers together.

Backwards problem - same as permutations but easier.

Pascal's Triangle

Notice patterns.
The outer numbers are always 1.
Each number is the sum of the 2 above numbers.
Row 'n' has 'n' values.

${}_nC_r$ has the same value as the $(n+1)^{th}$ row and the $(r+1)^{th}$ number in that row.

The Binomial Theorem

$$(x+y)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \dots + \binom{n}{n-1}xy^{n-1} + \binom{n}{n}y^n$$

$$(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{3!}x^{n-3}y^3 + \dots$$
$$+ nxy^{n-1} + y^n$$

eg) Substitution for $(3a-2b)^4$ $\quad x = 3a \quad y = -2b$

$$= \binom{4}{0}x^4 + \binom{4}{1}x^3y + \binom{4}{2}x^2y^2 + \binom{4}{3}xy^3 + \binom{4}{4}y^4$$
$$= (3a)^4 + 4(3a)^3(-2b) + 6(3a)^2(-2b)^2 + 4(3a)(-2b)^3 + (-2b)^4$$
$$= 81a^4 - 216a^3b + 216a^2b^2 - 96ab^3 + 16b^4$$