Dividing Polynomials
- used for division (quotients and remainders)
- used for simplification and reduction
- if remainder is zero, we know the divisor is a factor
- this is used in calculus
- pad missing powers with zeros. (placeholders)
- performed like long division with numbers
- division statement: \( P(x) = D(x) \cdot Q(x) + R(x) \)

\[
\begin{align*}
6x^4 + 8x^3 - 5x + 3 \\
\underline{2x^2 + 0x - 4} \bigg| 6x^4 + 8x^3 + 0x^2 - 5x + 3
\end{align*}
\]

Division Statement:

Watch for negative signs in subtraction!

Special Case: Monic Linear Binomial -
- if \( a \), stick with long division.
- use synthetic division (leave out variable and leading term)
- remember to still use zero placeholder for missing powers
- “\( a \)” so that we can add instead of subtract.

\[
(2x^3 + 3x^2 - 17x + 4) \div (x+4)
\]
Regular \[\frac{2x^2 - 5x + 3}{x + 4}\]

Synthetic \[\frac{-2x^3 + 3x^2 - 17x + 4}{-2x^2 + 8x^2}\]

\[\begin{array}{c}
2x^2 - 5x + 3 \\
-2x^3 + 8x^2 \\
-5x^2 - 17x \\
-5x^2 - 20x \\
3x + 4 \\
3x + 12 \\
8
\end{array}\]

\[\text{Synthetic is shorter, but is abstract.}\]

\text{Division Statement:}

\[\text{Special Case: Any linear binomial -}
\text{change to divisor to}
\text{divide intermediate quotient by}
\]

\[\text{eq.) } \frac{-6x^3 + 33x - 10}{-3x + 6}\]

\[\begin{array}{c|cccc}
2 & -6 & 0 & 33 & -10 \\
-12 & 0 & 33 & -10
\end{array}\]

\text{Answer:}

\text{HW: pp. 7-12: 3, 4, 6, 7, 8, 10, 11, 14}
\text{Challenge: 13, } (-3x^3 + 8x^2 - 10x + 5) \div (-x+2)
\text{Please answer 7b in person.}

\[\text{(2)}\]
Factor Polynomials - more powerful tools to factor

**The Remainder Theorem** - must be dividing by a monic linear binomial - when \( P(x) \) is divided by \( x - a \), the remainder is \( P(a) \).

eq) \( P(x) = 2x^2 - x - 1 \), remainder when dividing by \( x - 2 \)

\[
P(2) = 2 \quad 2 \quad -1 \quad 1 \]

```
    2 | 2 2 -1 1 1
  _____________
    2 2 -1 1
```

Factoring: \( 2x^2 - x - 1 \)

\[
P(1) = 2(1) - (1) - 1 = 0
\]

\[
P(-\frac{1}{2}) = 2 \left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right) - 1
\]

\[
= 2 \left(\frac{1}{4}\right) + \frac{1}{2} - 1
\]

\[
= \frac{1}{2} + \frac{1}{2} - 1
\]

\[
= 0
\]

If \( a \) is a zero of \( P(x) \), it should be no surprise that the value of \( P(a) = 0 \). This is the Factor Theorem:

\( x - a \) is a factor of \( P(x) \) if \( P(a) = 0 \).

Recall: PFS for factoring quadratic trinomials: the only numbers that we check are factors of .

So, the Factor Property states that if \( x - a \) is a factor of \( P(x) \), then \( x - a \) must be a factor of the constant term in \( P(x) \).

**Tip:** Once you find a factor, use the factor property for the new quotient.

**Tip:** Try smallest factors first, then increase in size.

\( \text{eq) For constant term 24:} \)
Example: \( P(x) = x^4 + 2x^3 - 13x^2 - 14x + 24 \)
-2 is a factor of 24

\[
P(-2) = (-2)^4 + 2(-2)^3 - 13(-2)^2 - 14(-2) + 24
= 16 - 16 - 52 + 28 + 24 = 0
\]

So \( x+2 \) is a factor

| -2 | 1  | 2  | -13 | -14 | 24 |

Tip: Intermediate Value Theorem

Backwards Problem: What value of \( k \) will result in \((2x^4 + kx^3 + kx^2 - 8) \div (x-3)\) having a remainder of 10?
\[ P(x) = 2x^4 + kx^3 + kx^2 - 8 \]

**Tips:**
If you are given values, it is best to substitute and see where an answer might come out.

There are 3 main tools in math: algebra, substitute, and factor.

**HW:** pp. 20 - 26: 3-6, 8, 9, 11, 12
Challenge: 13 - 15
Please answer in person: Why don't we try "0" as a factor?
We can use our graphing calculators to make some conjectures about higher degree polynomial functions. But it's faster to use Desmos. You still need to learn how to use a graphing calculator!

Let's look at even degree polynomials. The degree is the exponent of the largest power in a polynomial. E.g., the degree of $3x^2 + 5x^3 - 6x^4 + 30$ is 4.

- How does the degree affect the shape?
- How does the leading coefficient affect the shape?
- Do repeated roots have an effect?
- How would you describe the range?

Let's look at odd degree polynomials.

- How does the degree affect the shape?
- How does the leading coefficient affect the shape?

One thing that helps when examining the shape is to adjust the window. Sometimes you will need to zoom in and other times you will need to zoom out.
Tip: Start domain on calculator as $[-|a_0|, |a_0|]$ where $a_0$ is the constant term.
Sometimes the range also needs to be $[-|a_0|, |a_0|]$.

Question: Think about the number of hills and valleys and how they relate. Is it possible to have 2 hills and 1 valley?
2 hills and 2 valleys? 1 hill & 2 valleys? 3 hills and 1 valley?

Question: If the degree is 5, can we have 2 hills & 2 valleys? 1 hill & 2 valleys? 3 hills & 3 valleys?

HW: pp. 34-36: all, but answer 1c and 2b in person.
Mathematics

Relating Polynomial Functions and Equations.

The term with the highest power has the degree of the polynomial. It is the sum of all variable exponents.

\( x^5 \) - degree
\( x^2 y^3 \) - degree
\( x^2 y^3 + 2^6 y^4 \) - degree

The leading coefficient is the coefficient of the term with the highest power.

\(-3x^4 + 2x^2\) - leading coefficient is \(-3\)
\(-3x^3 + 4x^4 y^2\) - leading coefficient is \(4\)

We classify polynomials as even-degree and odd-degree.

From last lesson, where do even-degree polynomials finish when they start in quadrant II? quadrant III?

\(-x^2\) - \(-\)
\(-x^3\) - \(-\)

Where do odd-degree polynomials finish when they start in quadrant II? quadrant III?

\(-x^2\) - \(-\)
\(-x^3\) - \(-\)

Polynomials generally have peaks and valleys. This means we can have global/local minimums and maximums. The lowest point is the global minimum and the highest point is the global maximum.
Maximum number of min/max:

Question: Can an odd-degree polynomial have a global minimum/maximum?

Multiplicity - the number of times a zero repeats. Recall: zeroes for functions, roots for equations. We classify multiplicity as odd or even.

\[ y = (x-2)^2(x+1) \]

\[ y = (x+3)^4(x-4)^3 \]

Even multiplicity will have the curve touch but not cross the x-axis.
Odd multiplicity will have the curve cross the x-axis.

Use these properties and definitions to help you graph.

\[ y = -(x+5)^2(x+1)(x-2) \quad y_{\text{int}} = \]

Odd/Even

Graphing tips:
- use intercepts
- smaller peaks & valleys when zeros are closer
- polynomials are smooth, no jagged edges
- factor before sketching

HW: pp 46-53: 3, 4, 5, 7, 8, 9, 11, 13
Challenge: 14, 15
Please answer 126 in person.
Word problems generally require a good understanding of English. It also requires knowledge about a secondary subject like physics, economics, chemistry, business, etc.

Create a meaningful variable (looking at the question is a good place to start), since we are using polynomials, we only want one variable.

Per, each, portion, divvy, group, partition – usually mean divide (sometimes multiply)
Times, magnify - multiply
Is, will be, was – mean equal
More, and, plus, increase, sum, total, tally – usually mean add
Less, decrease, reduce, take away, deduct, discount – usually mean subtract
A dictionary or thesaurus can help with interpreting some words.

Many word problems rely on physics. Here are the basic physics formulas.
Distance = \( \frac{1}{2} \) acceleration*time\(^2\) + initial velocity*time + initial position
Average Velocity = total distance / time
Time = total distance / average speed

Here are the business formulas:
Average cost = total cost / number of units
Revenue = number of units * cost per unit
Profit or loss = Revenue – cost

Finally, look at the final question to determine what the variable should be. This will ensure that you don't have to do any further calculations to get the answer.
Example: A piece of cardboard 28 cm x 21 cm is used to make a box with no lid. What height will maximize volume? What height will make the volume 750 cm$^3$?

\[ V = l \cdot w \cdot h \]

Use graphing calculator:

BE CAREFUL with box problems. There are different set-ups.
Example: The school is raffling off an iPad worth $700. They know that they will sell 500 tickets if the price is $5. For each dollar they increase the price above $5, they will lose \( 10x^2 \) where 'x' is the dollar increase. Ticket prices must be in dollars. How many tickets sold?

\[ \text{Revenue} = \text{price} \cdot \# \text{ of tickets} \]
Definition: Polynomial Expression - 
A polynomial of degree \( n \) can be written in standard form as:

\[ a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_1 x + a_0 \]

where \( n \) is a whole number and \( a_n, \ldots, a_0 \) are 

The theorems in this chapter apply to polynomials with integer coefficients. Polynomials can have real value coefficients.

Example:

\[ -5x^4 + 2x^2 + 3 \]
\[ 7x^3 - 4x^2 - 1 \]

Not polynomial expressions:

\[ 3 \sqrt{x} - 4 + 5x^2 \]
\[ \frac{4}{x^2} + 3x^3 \]

Not standard form:

\[ x + 3 + 4x^3 \]
\[ 7 - 2x - 14x^4 \]

HW: pp. 61-66: 3-7, 9, 11
Challenge: 10, 12
Please answer 8 in person.