Properties of Radical Functions (specifically $\sqrt{x}$)

**Domain:**

**Range:**

Invariant Points occur where

Example:

$$y = \sqrt{x}$$

If $x = \frac{1}{4}$ then $\sqrt{x} = \sqrt{\frac{1}{4}} = \frac{1}{2}$.

So

In fact, for all $0 < x < 1$,

This means between 0 and 1, for $x$, we plot $\sqrt{x}$

If $x = 4$ then $\sqrt{x} = \sqrt{4} = 2$

So

In fact, for all $x > 1$,

This means that for $x > 1$, we plot $\sqrt{x}$

This is true no matter what $f(x)$ we choose to plot $f(\sqrt{x})$. $f(x)$ could be a quadratic, cubic, or any random function.
The properties of $\sqrt[n]{x}$ apply to all even root plots such as $\sqrt[2]{x}$ and $\sqrt[4]{x}$.

The properties of odd root plots are slightly different. We only consider $\sqrt[3]{x}$ for this chapter.

**Domain:**

**Range:**

Invariant Points occur at

**Example:**

For all $-1 < f(x) < 0$,
This means between $-1$ and $0$ for $f(x)$, we plot $\sqrt[3]{f(x)}$.

For all $f(x) < -1$,
This means that for $f(x) < -1$, we plot $\sqrt[3]{f(x)}$.

For a more accurate plot, use a table of values.
For $\sqrt[n]{f(x)}$, to find the domain, you need to solve $f(x) \geq 0$. This generally means solving a quadratic or cubic inequality.

From Pre-Calc II, we have quadratic inequalities, so we must consider the outer intervals or just the inner.

**Example:**

$y = \sqrt{x^2 + 4}$

**Discriminant:** $b^2 - 4ac = 0 - 16 < 0$

**Domain:**

**Range:**
eg 8) \[ y = -\sqrt{x^2 - 9} \]
\[ x^2 - 9 \geq 0 \]
\[ (x + 3)(x - 3) \geq 0 \]
\[ +a, \text{ so outer} \]

Domain:

Range:

eg C) \[ y = \sqrt{4 - x^2} \]
\[ 4 - x^2 \geq 0 \]
\[ x^2 - 4 \leq 0 \]
\[ (x + 2)(x - 2) \leq 0 \]
\[ +a, \text{ so inner} \]

Domain:

Range:

Tip for graphing calculators. \[ y_1 = f(x), \ y_2 = \sqrt{f(x)} \]

eg)
\[ f(x) = 2x - 4 \]
\[ f(x) = 5x + 3 \]
\[ f(x) = x^2 + x - 2 \]

Graphing without calculator: You know how to plot lines, quadratics, and cubics, so plot them and use root graphing rules.

HW: pp. 89-96: 1, 3, 4, 5, 7, 8, 9, 11, 13
Challenge: 10, 14
Please show 12b in person.
Graphing Rational Functions

Rational expressions have the form \( \frac{P(x)}{Q(x)} \) where \( P(x) \) and \( Q(x) \) are polynomial expressions.

Definitions

Vertical Asymptotes (V.A.) occur at the zeroes of \( Q(x) \). How many V.A.'s are possible if \( Q(x) \) has degree \( n \)?

When does a rational function have a horizontal asymptote (H.A.)?

Challenge: when does a rational function have a slant asymptote (S.A.)?

How can you tell what happens at the ends of the curve?

Definition Hole: is when a point is removed from a curve.
eg)

When do holes occur?
Can they occur with V.A.'s?

HW: pp. 101-104: A, B, 1, 2, 3a.
Please state what 3b has in person.

Group I: \( y = \frac{x^2 - x - 2}{x - 2} \)

Group II: \( y = \frac{1 - x^2}{x^2 + x - 6} = \frac{(1+x)(1-x)}{(x+3)(x-2)} \)

Group III: \( y = \frac{3x}{x^2 - x - 6} = \frac{3x}{(x-3)(x+2)} \)
Analyzing Rational Functions. \( \frac{P(x)}{Q(x)} \) domain? \( x \in \mathbb{R} \) except \( 0 \)’s

Where are the vertical asymptotes or holes?
At the zeroes of

When do you have holes vs. V.A.?
You get holes where you cancel out a factor with \( P(x) \), otherwise it’s a

When do you get a H.A.?
When the degree of \( Q(x) \) is \( \geq \) degree \( P(x) \).
If \( \text{deg}(Q(x)) = \text{deg}(P(x)) \), then \( H.A. \)
Where \( a \) is the L.C. of \( P(x) \) and \( b \) is the L.C. of \( Q(x) \).
Otherwise \( H.A. \) is

When do you get a S.A. (slant or oblique asymptote)
When \( \text{deg}(P(x)) = \text{deg}(Q(x)) + 1 \) and there
is a V.A.
You can calculate the S.A. by polynomial division.

\[
\begin{align*}
\text{eg:} & \quad \frac{2x^2 + 3x + 5}{x + 3} \\
& \quad \begin{array}{c|ccccc}
-3 & 2 & 3 & 5 \\
\hline
 & 2 & 3 & 5 \\
\end{array}
\end{align*}
\]

S.A.

\[
\text{out} \quad \frac{(x - 3)(x + 1)}{x - 3}
\]

\[
\text{eg:} \quad \frac{(x + 4)(x - 5)(x + 2)}{(x - 4)(x + 2)}
\]
Some quick examples:

\[ \frac{(x+2)(x+3)}{(x+2)(x-1)(x-4)} \]

\[ \frac{4x^2+3x+9}{2x^2+4x+7} \]

\[ \frac{x^2+x-6}{x+1} \]

\[ \frac{(x+4)(x-3)(x-5)(x+2)}{(x+1)(x-4)} \]
Sketching Graphs of Rational Functions. \( \frac{P(x)}{Q(x)} \)

1. Use **N.D.V.s** to find V.A.'s and holes.
   Holes are at common factors between \( P(x) \) & \( Q(x) \)

2. H. A.'s occur when \( \deg (P(x)) \leq \deg (Q(x)) \)
   \[ y = \frac{a}{b} \] where \( a \) & \( b \) are L.C. of \( P \) & \( Q \) if
   \[ \deg (P(x)) = \deg (Q(x)) \]
   \[ y = 0 \] if \( \deg (P(x)) < \deg (Q(x)) \)

3. S. A.'s occur when \( \deg (P(x)) = \deg (Q(x)) + 1 \) and V. A.
   Use polynomial division for S. A.

4. Find behaviour near V. A.'s (generally \( \pm 1 \) or \( \pm 0.1 \) away).

5. Find **x-intercepts** (the zeroes of \( P(x) \)).

6. Find the y-intercept (set \( x = 0 \) and solve for \( y \)).

7. Label all intercepts.

8. Join points smoothly.

**eg.** Sketch \( f(x) = \frac{x^2}{(x-2)(x-4)} \)

1. NO V.V.S: \( x=2,4 \),
2. \( \deg (x^2) = \deg (x^2 - 6x + 8) \),
3. No S.A.
4. \begin{align*}
   & \text{at } x = 1.9, \\
   & \text{at } x = 2.1, \\
   & \text{at } x = 3.9, \\
   & \text{at } x = 4.1,
\end{align*}
5. **x-intercepts:**
6. **y-intercept:**
eg) Sketch \( f(x) = \frac{(x-3)(x+2)(x+4)}{(x+4)(x-4)} \)

1. \( \text{NPV's:} \ x = -4, 4 \)

2. \( \text{deg} (P(x)) \neq \text{deg} (Q(x)) \)

3. \( \text{deg} (P(x)) = \text{deg} (Q(x)) + 1 \)
   \((x-3)(x+2) = x^2 - x - 6\)

4. at \( x = 3, 9, \)
   at \( x = 4, -1, \)

5. \( x\)-intercepts:

6. \( y\)-intercept:
HW: pp. 134-139: 3a, 4a, 5, abd, b(i & ii)
Challenge: 8 ab
Please show 4b in person
Do without calculator!
There is a non-calculator section on the test!