Chapter is transforming graphs.

\[ y = a f(b(x-h)) + k \] is a transform of \( f(x) \).

- Translation
- Scaling (reflected if negative)

We start with translations.

\[ y = f(x-h) + k \quad \text{or} \quad y - k = f(x-h) \]

The graph of \( f(x) \) is translated \( h \) units horizontally and \( k \) units vertically.

eg) Plot \( g(x) = f(x-3) + 2 \)

The easiest way to translate a function:
- Plot the fn \( f(x) \)
- Translate key points by \((h, k)\)
- Copy curves between key points.
- \( h \), positive is right, negative is left
- \( k \), positive is up, negative is down
Determining the Translation from Descriptions

e.g.) Translate \( f(x) \) down 3 and right 2 as \( g(x) \).

e.g.) Translate \( f(x) \) left 5 and up 4 as \( g(x) \).

Determining Translation from Graphs

Find a feature, if not given, to determine the translation from corresponding points. Use minimums or maximums or grid points if no corresponding points given.

e.g.) Write \( g(x) \) as a translation of \( f(x) \).

Definition - Image: is the function after the transformation. (just the given interval)

Definition - PreImage: is the function before the transformation. (just the given interval)

e.g.) The following is the graph of the image of a translation of right 2 and up 3. Draw the preimage.

HW: pp. 169-175: 4-6, 8, 9, 11b, 12, 14, 15

Challenge: 17, 18

Please graph 11a in person.
Chapter is transforming graphs.

\[ y = af(b(x-h)) + k \] is a transform of \( f(x) \).

- Reflecting:
  - \( y = -f(x) \) reflection on
  - \( y = f(-x) \) reflection on

**Example:** Plot \( g(x) = f(-x) \)

- Domain of \( f \):
- Range of \( f \):
- Domain of \( g \):
- Range of \( g \):

In general: negate domain, range is unchanged.
If \( f(x) = f(-x) \), we call this function \emph{even}. An example is

**Example:** Plot \( g(x) = -f(x) \)

- Domain of \( f \):
- Range of \( f \):
- Domain of \( g \):
- Range of \( g \):

In general, negate range, domain unchanged.
eg) Plot \( g(x) = -f(-x) \)

\[\text{D of } f:\]
\[\text{R of } f:\]

\[\text{D of } g:\]
\[\text{R of } g:\]

In general: negate domain and range.
If \( f(x) = -f(-x) \), then we say the function is \( f(x) \).
An example is \( f(x) \). To test,

The easiest way to reflect a function:
- plot the \( f(x) \)
- reflect key points on x and/or y-axis
- draw curves between key points.

eg) Reflect \( f(x) = (x-2)^3 - 4 \) on the y-axis. Find \( g(x) \) for this reflection.

Where does \( f(x) = g(x) \)? always?

eg) Reflect \( f(x) \) on the x-axis. Find \( h(x) \) for this.

Where does \( f(x) = h(x) \)? always?

HW: pp. 184 - 190: 4, 6 - 10, 12
Challenge: 13, 14
Please show 11 in person.
Chapter is transforming graphs.

\[ y = af(b(x-h)) + k \] is a transform of \( f(x) \).

- **Scaling**
  - \( y = af(x) \) if \( |a| < 1 \), vertical
  - \( y = af(x) \) if \( |a| > 1 \), vertical

**Example 1:** Plot \( g(x) = 2f(x) \)

- **Domain of \( f \):**
- **Range of \( f \):**
- **Domain of \( g \):**
- **Range of \( g \):**

In general, domain unchanged, range is multiplied by \( a \).

\[ y = f(bx) \] if \( |b| < 1 \), horizontal
\[ y = f(bx) \] if \( |b| > 1 \), horizontal

**Example 2:** Plot \( g(x) = f(5x) \)

- **Domain of \( f \):**
- **Range of \( f \):**
- **Domain of \( g \):**
- **Range of \( g \):**

In general, domain is divided by \( b \), range is unchanged.
We can combine horizontal and vertical scaling.

\( g(x) = -1.5 f(-2x) \)

\( f \) of \( f \):
\( R \) of \( f \):

\( g \) of \( g \):
\( R \) of \( g \):

The easiest way to scale a function:
- plot the \( f \)n \( f(x) \)
- Scale key points
- Copy curves between key points.

Transforming preimage to image using \( a \) & \( b \):
\((x, y) \rightarrow (x', y')\)

Transforming image to preimage using \( a \) & \( b \):
\((x', y') \rightarrow (x, y)\)

eg) The point on the image is \((5, 2)\) and the transform is a vertical comp. of 3 and horizontal stretch of 2. Find the preimage point.

eg) The preimage has a point, \( P = (10, 6) \). The image has a corresponding point, \( P' (5, 2) \). Determine \( 'a' \) & \( 'b' \).

HW: pp. 201 - 209: 3, 4, 5, 7, 8, 9, 11, 13
Challenge: 15, 16
Please show 12 in person.
Chapter 3.4

Chapter is transforming graphs.

\[ y = a f(b(x-h)) + k \]

is a transform of \( f(x) \).

Combining all transformations - How to plot:
- plot the fn \( f(x) \)
- apply stretching/compression for horiz/vert.
- apply horiz/vert. reflections
- apply translations.

If you reverse the order, it will be incorrect! Why?

You can do all transforms for each point or each transform on \( f(x) \).

**eg**

\[ f(x) = x^2 \quad & \quad g(x) = f(\frac{1}{2}(x-1)) \]

Transform preimage, \( P = (x, y) \), to image, \( P' = (x', y') \):

Transform image \( P' \) to \( P' \).
The same applies for transforming the domain and range:  

- Domain of $g$: \( (\text{Domain of } f) / b + h \)  
- Range of $g$: \( a(\text{Range of } f) + k \)  

Domain of $f$:  
Range of $f$:  

Domain of $g$:  
Range of $g$:  

Note: For a very few graphs, there are no well-defined corresponding points to transform. So you can pick reasonable ones. An example of this is exercise 3 on p. 226.
Determine the transformation given the preimage points, \( A = (-5, -6) \) & \( B = (-1, 6) \) and the corresponding points on the image, \( A' = (-13, 17) \) & \( B' = (-1, -7) \). Determine the transformation function, \( g(x) \), as a function of \( f(x) \).

Question could also be phrased as what the transformations are or what are the constants.

HW: pp. 226 - 232: 1, 3, 4, 5, 7, 8a, 9, 11
Challenge: 12
Please show 8b in person.
Inverse Relations (as opposed to functions)

If given a graph, reflect curve on $y=x$. Can also take key points, swap $x$ & $y$; plot; then mirror curve between the key points.

Example:

- Domain of $f$: $-4 \leq x \leq 2$
- Range of $f$: $-1 \leq y \leq 3$
- Domain of $g$:
- Range of $g$:

In general: swap domain and range for $g$.

Can check reflection by drawing a line between corresponding key points and check that they are equi-distant and perpendicular to

Necessary for full marks.

Inverse done algebraically:

Example 1:

- $y = 3x + 7$
- Swap $x$ & $y$
- Solve for $y$
- Inverse function

Example 2:

- Are $y = 2x - 4$ and $y = \frac{1}{2}x + 6$ inverses of each other?
eg) \[ y = \frac{2x - 9}{3x - 5} \]

- Swap x & y
- Solve for y
- Multiply denom to LHS & expand
- Move all y terms to LHS
- Factor out y
- Isolate y

eg) \[ y = (x-3)^2 + 5 \]

- Swap x & y
- Isolate y term
- Square root of both sides
- Isolate y

Inverse is a function if it passes the vertical line test. However, we can tell if there is an inverse function by doing a horizontal line test on the original function.

HW: pp. 243-248: 4 - 6, 7ab, 8, 9, 10a, 12, 13
Challenge: 14 - 16
Please show 11 in person.