5.1) Exponentials - $a^x$ variable is in the exponent
- base, 'a' is constant not a var
- $a > 0, x \in \mathbb{R}$, Range > 0
- When $a > 1$, exp grows - grows faster than polynomials.
- $0 < a < 1$, exp decays
- HA: $y = 0$ (untransformed)

5.2) Transforms: $f(x) = a^x$

$y(t) = y_0 \cdot r^{kt}$

Y-intercept

$y(t) = y_p$ (some point on the curve)

$y - y_0 = m(x - x_0)$

$(3, 12)$ Given $r = 1.5$ $k = 1$

$y(t) = 12 (1.5)^{t-3}$

- $y(t), y_0$ it's okay to be a percentage.
- If $r$ is given as a percent, it needs to be changed to a number. (ie $\div 100$)
- Important: When given a rate, determine if it is a change (by) or a value (to). If (by) then add or subtract from 100%

5.3) Solving

- make base the same, then equate exponents
- use logs or use calculator - beware of x-int or intersection
- avoid round intermediate calculations
- do all algebra then enter into calculator.
5.4) Logarithms are used to determine exponents, i.e. a variable in the exponent. \[ C = b^a \iff \log_b C = a \]
\[ y = \log_b x \quad \text{D: } x > 0 ; \quad R: y \in \mathbb{R}, \quad b > 0 \land b \neq 1 \]
\[ \forall a: x = 0 \quad \text{(untransformed)} \]
Changing base: \[ \log_b 70 = \frac{\log 70}{\log 3} \]
Do linear interpolation instead of benchmarks.

5.5) Log Laws:
\[ \log_a x + \log_a y = \log_a xy \quad \log_a a = 1 \]
\[ \log_a x - \log_a y = \log_a \frac{x}{y} \quad \log_a a^x = x \]
\[ \log_a x^k = k \log_a x \quad \log_b x = \frac{\log_a x}{\log_a b} \]
Determine log if given some log values.
- factor out 10 first, then factor out given logs.
- recall \( \log 10 = 1 \)

5.6) Transforms:
\[ f(x) = \log_a x \quad y = c f(d(x-h)) + k \]
\[ y = c \log_a (d(x-h)) + k \]
Easiest point to transform: \( x\text{-int } \rightarrow \log_a 1 = 0 \quad (1, 0) \)

5.7) Solving Exp & Log Equations.
- Check logs for extraneous.
For exp eqns with no common base or constants, use logs to solve for exact answers. eg) \( 5^x = 150 \)
For log eqns with constants, simplify logs first, then use exp to solve for exact answers.
\[ \text{eg) } \log_2 x + \log_2 (x+3) = 4 \]
\[ \log_2 (x^2 + 3x) = 4 \]
\[ x^2 + 3x = 2^4 \]
5.8) Financials - choose proper eqn.

**Compound Interest (lump sum at beginning):**

\[ A(t) = A_0 (1 + \frac{i}{n})^{nt} \]

- \( i \) - rate (percent/100%)
- \( n \) - # of compoundings
- \( t \) - years.

\[ i = \frac{\text{percent}}{100} \times \text{# compounding periods} \]
\[ n = \text{# of payments} \]
\[ \text{years} = \frac{n}{\text{# compoundings}} \]

**Future Value (no lump sum):**

\[ FV = PMT \left( \frac{(1+i)^n - 1}{i} \right) \]

**Present Value (loan):**

\[ PV = PMT \left( \frac{1 - (1+i)^{-n}}{i} \right) \]

Check answer: \( PMT \cdot n > \text{loan amount} \)
\( PMT \cdot n < \text{Total savings} \)

**Earthquakes:**

\[ M = \log \left( \frac{I}{I_0} \right) \]

\( I \) - intensity should be some number of times \( I_0 \).

\( I_0 \) - constant magnitude.

**Decibels:**

\[ L = 10 \log \left( \frac{I}{I_0} \right) \]

\( L \) - intensity should be some number of times \( I_0 \).

**Alkalinity:**

\[ pH = -\log (a_{H^+}) \]

\( a_{H^+} \) - hydrogen concentration.

Lower pH is more acidic.
Higher pH is more alkaline.

\( 3.4 < 3.7 \) so more acidic.

A sound is \( \frac{1}{4} \) quieter than 68 dB, what is the decibels of that sound.

\[ L = 10 \log \left( \frac{I}{I_0} \right) \]

\( 68 = 10 \log \left( \frac{I}{I_0} \right) \)

\[ 6.8 = \log \left( \frac{I}{I_0} \right) \]

\[ I = 10^{6.8} I_0 \]

\[ L = 10 \log \left( \frac{10^{6.8} I_0}{I_0} \right) \]

\( L = 10 \log (10^{6.8} I_0 / I_0) \)

\( 62 \) dB