6.1) Trig in Standard Position

When terminal sides line up, we say they are co-terminal
\pm 360°n, or \pm 2\pi n, n \in \mathbb{Z}

Pythagoras to determine missing side:
\( r = \sqrt{x^2 + y^2} \)
\( x = \sqrt{r^2 - y^2} \)
\( y = \sqrt{r^2 - x^2} \)

Use ASTC to determine sign of \( x \) & \( y \), \( r \) is always positive.

On unit circle (\( r = 1 \)):
\[ y = \sin \theta \]
\[ x = \cos \theta \]
\[ \tan \theta = \frac{y}{x} = \text{slope} \]

Reciprocal Trig:
\[ \csc \theta = \frac{1}{\sin \theta} = \frac{1}{y} \]
\[ \sec \theta = \frac{1}{\cos \theta} = \frac{1}{x} \]
\[ \cot \theta = \frac{1}{\tan \theta} = \frac{x}{y} \]

Reference Angles:
(Use prime notation ')

\[ 0^\circ \leq \theta \leq 90^\circ \]
or
\[ 0 \leq \theta \leq \frac{\pi}{2} \]

I:
\[ \theta = \theta' \]
\[ \theta' = \theta \]

II:
\[ \theta = 180^\circ - \theta' \]
\[ \theta' = 180^\circ - \theta \]

III:
\[ \theta = 180^\circ + \theta' \]
\[ \theta' = \theta - 180^\circ \]

IV:
\[ \theta = 360^\circ - \theta' \]
\[ \theta' = 360^\circ - \theta \]

Solving trig with box. Find (box') & solve ASTC
quadrants, the divide all answers by 'b'.

eg: \( \text{solve } \sin 3x = -\frac{\sqrt{2}}{2} \)

\[ (3x)' = \sin^{-1} \left( -\frac{\sqrt{2}}{2} \right) = \frac{7\pi}{18} \]

III
\[ 3x_1 = \pi + \frac{7\pi}{6} \]
\[ 3x_1 = \frac{13\pi}{6} \]
\[ x_1 = \frac{7\pi}{18} \]

IV
\[ 3x_2 = 2\pi - \frac{7\pi}{18} \]
\[ 3x_2 = \frac{29\pi}{18} \]

Wrong: \( x_2 = 2\pi - \frac{7\pi}{18} = \frac{25\pi}{18} \)
6.2) Arc length \( s = r \theta \)  
\( \theta = \frac{\text{degrees}}{180^\circ} \)  
\( r = \frac{\text{radians}}{\pi} \)  
\( \text{radians} \times \frac{\pi}{180^\circ} \)

Degrees = radians. 
Radians = degrees \( \frac{\pi}{180^\circ} \)

State general solutions use Principal Angles: 
(unless told otherwise): 
\( 0^\circ \leq \theta < 360^\circ \quad 0 \leq \theta < 2\pi \)
or just think smallest positive coterminal angle.

6.3) Special Angles

<table>
<thead>
<tr>
<th>Reference</th>
<th>( \sin \theta )</th>
<th>( \cos \theta )</th>
<th>( \tan \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>30°</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{\sqrt{3}}{2} )</td>
<td>( \frac{\sqrt{3}}{3} )</td>
</tr>
<tr>
<td>45°</td>
<td>( \frac{\sqrt{2}}{2} )</td>
<td>( \frac{\sqrt{2}}{2} )</td>
<td>1</td>
</tr>
<tr>
<td>60°</td>
<td>( \frac{\sqrt{3}}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>( \sqrt{3} )</td>
</tr>
<tr>
<td>90°</td>
<td>1</td>
<td>0</td>
<td>undefined</td>
</tr>
</tbody>
</table>

Area of sector: \( A = \theta \) \( \frac{(\pi r^2)}{360^\circ} \)

Angular velocity: \( V_a = \frac{\Delta \theta}{t} \)

Distance = \( V_a t \)

Revolution: \( 1 \text{ rev} = 2\pi \text{ radians} \)

6.4) Inverse Trig - do on \( |\text{ratio}| \) - gives reference angle, \( \theta \).

Then determine quadrant and calc \( \theta \).

Period - length of cycle to repeat: \( \sin \theta \) & \( \cos \theta \) - \( 2\pi/n \)

\( \tan \theta = n/b \)

6.5) Properties -  
center line: \( d = \frac{\text{max} + \text{min}}{2} \)  
amplitude: \( a = \frac{\text{max} - \text{min}}{2} = \text{max} - \text{center line} \)  
phase shift (c): S-measure c.l. going up for sine  
from y-axis < -measure max for cosine

6.6) Graphing:  
Draw dotted center line, max, min. 
Draw point for max phase shifted with cosine \( c.l. \) going up phase shifted with sine  
Draw points for cycles using the period 
Draw cycles.
Word Problems:

radius ≡ amplitude
diameter = 2 (amplitude) ≡ peak-valley = max-min
revolution ≡ full circle = cycle = 360° = 2π radians
min = low = least = valley
max = high = most = peak
center line ≡ mean = avg ≡ center = middle = axle height