8.1-8.4) Fundamental Counting Principle
Permutations (no repeat) - order matters
Permutations (with repeats) -
Combinations - order doesn’t matter

Deciding which one:
- think about whether you are choosing from same or different sets - if diff, then FCP
- think about if we are selecting an exact number of items. If yes, then P or C
otherwise FCP.

Example 1: Pizza/Sandwich
We can select all or no toppings.
How many choices?
Pepperoni is a set of 2 choices
Not using exact # of items FCP
eg) 12 Basketball players
Opposing shake hands
How many hand shakes?
each team is own set (2)
FCP
12 \times 12 = 144

Example 2: How many ways can 14 IHIF hockey teams finish 1st to 4th in the playoffs?
One set
Order matters - P
14P4 = 2424

Example 3: How many ways can 6 sci-fi, 5 romance, and 7 horror novels be placed on a shelf grouped by genre?

Example 4: Your choice of 3 toppings
How many choices?
All toppings are a set.
Exact # of items (8)
Order doesn’t matter - C
eg) 12 Basketball players.
Same team shakes hands.
The team is one set.
Order doesn’t matter - C
12C2 = 66

Example 5: How many ways can 14 IHIF hockey teams qualify for 4 playoff spots?
One set
Order doesn’t matter - C
14C4 = 1001

Example 6: How many ways can we buy 2 novels from each genre...
Different sets - FCP

Exact # of items
Order matters - P
\[ \binom{5}{3} \times \binom{6}{2} \times \binom{5}{2} \times \binom{7}{1} = 26,128,800 \]

Order doesn't matter - C
Different sets - FCP

\[ \binom{5}{2} \times \binom{5}{2} \times \binom{7}{2} = 3,150 \]

by genre: Romance, Sci-fi, Horror

FCP

Reasoning

eg) 3 couples want to sit in a row with couples together. How many ways can they be seated?

arrange 3 couples = 3!

1st couple (2 seats) = 2!

2nd couple (2 seats) = 2!

3rd couple (2 seats) = 2!

FCP (different sets): 3! \times 2! \times 2! \times 2! = 48.

Alternately:

6 choices for 1st seat
no choices for 2nd seat.
4 choices for 3rd seat.
no choices for 4th seat.
2 choices for 5th seat.
no choices for 6th seat.

FCP: 6 \times 4 \times 2 = 48.

Backwards Problems

eg) Solve for \( n, \ nP_4 = 11,880 \)

\[ \frac{n!}{(n-4)!} = n(n-1)(n-2)(n-3) = 11,880 \]

\[ y_1 \leq y_2 \text{ window } x \in [0, 20] \]

\[ y \in [0, 13,000] \]

eg) Solve for \( r, \ 13P_r = 12,355,200 \)

\[ 13 	imes 13, 13 	imes 12 = 156, 13 \times 12 \times 11 = 1716 - - - \]

\[ 13 \times 12 \times 11 \times 10 \times 9 \times 8 \]

\[ r = 6 \]

eg) Solve for \( n, \ \binom{n}{4} = 330 \)

\[ \frac{n!}{(n-4)!4!} = 330 \]

\[ (n-4)!4! \times (n-4)! = 330 \]

\[ (n-4)!4! = 330 \]

\[ (n-4)!4! = 330 \times 4! \]

\[ (n-4)! = 330 \times 4 \]

\[ (n-4)!4! \times (n-4)! = 330 \]

\[ y_1 \leq y_2 \text{ window } x \in [0, 20] \]

\[ y \in [0, 13,000] \]

\[ \sqrt{120} = 11 \]
eg) Solve for \( r \), \( \binom{14}{r} = 1001 \)

\[ 14^2 = 196, \quad \frac{14 \cdot 13}{2} = 91, \quad \frac{14 \cdot 13 \cdot 12}{2 \cdot 3} = 364, \quad \frac{14 \cdot 13 \cdot 12 \cdot 11}{2 \cdot 3 \cdot 4} = 1001 \]

\[ r = 4. \]