PreCalc 12 Final Review Chp 8

Note Title 2016-05-27

8.1-8.4) Fundamental Counting Principle
Permutations (no repeat) - order matters
Permutations (with repeats) - "
Combinations - Order doesn't matter

Deciding which one.
- think about whether you are choosing from same or different sets - if diff, then FCP
- think about if we are selecting an exact number of items. If yes, then P or C otherwise FCP.

eg) Pizza/Sandwich
We can select all or no toppings.
How many choices?
Pepperoni is a set of 2 choices
Not using exact # of items
FCP

eg) Pizza/Sandwich
Your choice of 3 toppings
How many choices?
All toppings are a set.
Exact # of items (3)
Order doesn't matter - C

eg) 12 Basketball players
Opposing shake hands
How many hand shakes? →
each team is own set (2)
FCP
$12 \cdot 12 = 144$

eg) 12 Basketball players.
Same team shakes hands.
The team is one set.
Order doesn't matter - C
${}_{12}C_2 = 66$

eg) How many ways can 14 IIHF hockey teams finish 1st to 4th in the playoffs?
One set
Order matters - P
${}_{14}P_4 = 24024$

eg) How many ways can 14 IIHF hockey teams qualify for 4 playoff spots?
One set
Order doesn't matter - C
${}_{14}C_4 = 1001$

eg) How many ways can 6 sci-fi, 5 romance, and 7 horror novels be placed on a shelf grouped by genre?

eg) How many ways can we buy 2 novels from each genre ...

Different sets - FCP
Exact # of items
Order matters - P

$({}_3P_3)({}_6P_6)({}_5P_5)({}_7P_7) = 261273600$

↑ by genre | sci-fi | romance | horror

FCP

Order doesn't matter - C
Different sets - FCP

$\binom{6}{2}\binom{5}{2}\binom{7}{2} = 3150$

sci-fi | romance | horror

FCP.

Reasoning

eg) 3 couples want to sit in a row with couples together. How many ways can they be seated?

arrange 3 couples $= 3!$ ${}_3P_3$
" 1^{st} couple (2 seats) $= 2!$
" 2^{nd} " $= 2!$
" 3^{rd} " $= 2!$

FCP (different sets): $3!\,2!\,2!\,2! = 48$.

Alternately:

6 choices for 1^{st} seat
no choices for 2^{nd} seat.
4 " 3^{rd} "
no " 4^{th} "
2 " 5^{th} "
no " 6^{th} "

FCP $= 6 \cdot 4 \cdot 2 = 48$.

Backwards Problems

eg) Solve for n, ${}_nP_4 = 11880$

$\frac{n!}{(n-4)!} = \underbrace{n(n-1)(n-2)(n-3)}_{y_1} = \underbrace{11880}_{y_2}$

window $x \in [0, 20]$
$y \in [0, 13000]$

$n = 12$

eg) Solve for r, ${}_{13}P_r = 1235520$

$13 = 13$, $13 \cdot 12 = 156$, $13(12)(11) = 1716$ ...

$13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8$

$r = 6$

eg) Solve for n, $\binom{n}{4} = 330$

$\frac{n!}{(n-4)!\,4!} = 330 \Rightarrow \frac{n!}{(n-4)!} = 330(4!) \Rightarrow \underbrace{n(n-1)(n-2)(n-3)}_{y_1} = \underbrace{7920}_{y_2}$

$n = 11$

eg) Solve for r, $\binom{14}{r} = 1001$

$14 = 14$, $\frac{14 \cdot 13}{2} = 91$, $\frac{14 \cdot 13 \cdot 12}{2 \cdot 3} = 364$, $\frac{14 \cdot 13 \cdot 12 \cdot 11}{2 \cdot 3 \cdot 4} = 1001$

$r = 4$.